

On MHD transient flow of a Maxwell fluid in a porous medium and rotating frame

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Abstract

Analytic solution for unsteady magnetohydrodynamic (MHD) flow is constructed in a rotating non-Newtonian fluid through a porous medium. Constitutive equations for a Maxwell fluid have been taken into consideration. The hydromagnetic flow in the uniformly rotating fluid is generated by a suddenly moved infinite plate in its own plane. Analytic solution of the governing flow problem is obtained by means of the Fourier sine transform. It is shown that the obtained solution satisfies both the associate partial differential equation and the initial and boundary conditions. The solution for a Navier–Stokes fluid is recovered if $\lambda \rightarrow 0$. The steady state solution is also obtained for $t \rightarrow \infty$.

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1. Introduction

It has generally been recognized that in technological applications non-Newtonian fluids are more appropriate than Newtonian fluids. This is perhaps due to their demands in bio-rheology, geophysics, chemical and petroleum industries [1]. Because of the difficulty to suggest a single model which exhibits all properties of non-Newtonian fluids, they cannot be described as simply as Newtonian fluids. Due to this fact many models of constitutive equations have been proposed and most of them are empirical or semiempirical [2]. Amongst these the differential type fluid model gained considerable attention of many researchers. However on the otherhand there are much controversies on these models as well. Such fluids are also inadequate to describe the relaxation phenomena. For a complete

and detailed discussion of the relevant issues for differential type fluids, we refer the readers to Dunn and Rajagopal [3] and Aksel [4].

There is a subclass of the rate type fluids namely the Maxwell fluids in which the relaxation phenomena can be taken into account. Specifically the Maxwell fluid model has been used for the viscoelastic flows where the dimensionless relaxation time is small. However in some more concentrated polymeric fluids the Maxwell model is also useful for large dimensionless relaxation time. Some recent investigations dealing with the flows of Maxwell fluids are given in the references [5–10].

All the above investigations of hydrodynamic fluids are, however, confined to flows of Maxwell fluids in the non-porous medium and a non-rotating frame. In the present Letter our concern is to analyze the MHD rotating flow of a Maxwell fluid in a porous medium. The flow is caused by a suddenly jerked plate in its own plane. Fourier sine transform is employed to obtain the analytic solution. The obtained solution is valid for all times starting from the initial steady state ($t = 0$) to the final steady

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state ($t \rightarrow \infty$). The results have been compared with those of Singh and Sathi [11]. Finally, graphs of velocity are sketched and discussed.

2. Formulation of the problem

Let us consider an infinite non-conducting rigid plate (located at $z = 0$) and the incompressible Maxwell fluid (which is in contact with the plate) are in uniform rotation with angular velocity $\boldsymbol{\Omega} = \Omega \mathbf{k}$ ($\mathbf{k}$ is a unit vector parallel to the z -axis). The fluid occupies the porous medium $z > 0$. Moreover, the fluid is electrically conducting in the presence of an applied magnetic field $\mathbf{B}_0 = (0, 0, B_0)$. The magnetic Reynolds number is assumed small and hence the induced magnetic field is neglected. The Lorentz force $\mathbf{J} \times \mathbf{B}_0$ under these conditions is equal to $-\sigma B_0^2 \mathbf{V}$. Here $\mathbf{J}$ is the current density, σ is the electrical conductivity of fluid and $\mathbf{V}$ is the velocity field. Referred to a rotating frame of reference, the flow in a porous medium is governed by the following equations:

$$\text{div } \mathbf{V} = 0, \quad (1)$$

$$\rho \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} + 2\boldsymbol{\Omega} \times \mathbf{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) \right] = -\nabla p + \text{div } \mathbf{S} - \sigma B_0^2 \mathbf{V} + \mathbf{R} \quad (2)$$

in which ρ is the fluid density, $\mathbf{r}$ is a radial vector with $r^2 = x^2 + y^2$, p is the pressure and $\mathbf{R}$ is the Darcy's resistance. The extra stress tensor $\mathbf{S}$ for a Maxwell fluid satisfies

$$\mathbf{S} + \lambda \left[\frac{\partial \mathbf{S}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{S} - \mathbf{L} \mathbf{S} - \mathbf{S} \mathbf{L}^\top \right] = \mu \mathbf{A}_1, \quad (3)$$

where $\mathbf{L} = \text{grad } \mathbf{V}$, μ is the dynamic viscosity, λ is the relaxation time and $\mathbf{A}_1$ is the first Rivlin–Ericksen tensor defined by

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^\top. \quad (4)$$

According to Tan and Masuoka [12], the Darcy's resistance in an Oldroyd-B fluid satisfies the following expression

$$\left(1 + \lambda_r \frac{\partial}{\partial t} \right) \mathbf{R} = -\frac{\mu \phi}{k} \left(1 + \lambda_r \frac{\partial}{\partial t} \right) \mathbf{V}, \quad (5)$$

where λ_r is the retardation time, ϕ is the porosity and k is the permeability of the porous medium. For Maxwell fluid $\lambda_r = 0$ and hence

$$\left(1 + \lambda \frac{\partial}{\partial t} \right) \mathbf{R} = -\frac{\mu \phi}{k} \mathbf{V}. \quad (6)$$

We seek a velocity field of the form

$$\mathbf{V} = (u(z, t), v(z, t), w(z, t)) \quad (7)$$

which together with Eq. (1) yields $w = 0$. Therefore using Eqs. (2)–(4) and (7) we arrive at

$$\rho \left(\frac{\partial u}{\partial t} - 2\Omega v \right) = -\frac{\partial \hat{p}}{\partial x} + \frac{\partial S_{xz}}{\partial z} - \sigma B_0^2 u + R_x, \quad (8)$$

$$\rho \left(\frac{\partial v}{\partial t} + 2\Omega u \right) = -\frac{\partial \hat{p}}{\partial y} + \frac{\partial S_{yz}}{\partial z} - \sigma B_0^2 v + R_y, \quad (9)$$

where

$$\left(1 + \lambda \frac{\partial}{\partial t} \right) S_{xz} = \mu \frac{\partial u}{\partial z}, \quad (10)$$

$$\left(1 + \lambda \frac{\partial}{\partial t} \right) S_{yz} = \mu \frac{\partial v}{\partial z} \quad (11)$$

and the modified pressure $\hat{p}$ is

$$\hat{p} = p - \frac{\rho}{2} \Omega^2 (x^2 + y^2).$$

In Eqs. (8) and (9) R_x and R_y are the x - and y -components of the Darcy's resistance R and z -component of Eq. (2) indicates that $\hat{p} \neq \hat{p}(z)$. Invoking Eqs. (6), (10) and (11) in Eqs. (8) and (9) and then neglecting the pressure gradient we get

$$\begin{aligned} \rho \left(1 + \lambda \frac{\partial}{\partial t} \right) \left(\frac{\partial u}{\partial t} - 2\Omega v \right) \\ = \mu \frac{\partial^2 u}{\partial z^2} - \sigma B_0^2 \left(1 + \lambda \frac{\partial}{\partial t} \right) u - \frac{\mu \phi}{k} u, \end{aligned} \quad (12)$$

$$\begin{aligned} \rho \left(1 + \lambda \frac{\partial}{\partial t} \right) \left(\frac{\partial v}{\partial t} + 2\Omega u \right) \\ = \mu \frac{\partial^2 v}{\partial z^2} - \sigma B_0^2 \left(1 + \lambda \frac{\partial}{\partial t} \right) v - \frac{\mu \phi}{k} v. \end{aligned} \quad (13)$$

The appropriate initial and boundary conditions are

$$u = v = 0 \quad \text{at} \quad t = 0, \quad z > 0, \quad (14)$$

$$u(0, t) = U_0, \quad v(0, t) = 0 \quad \text{for} \quad t > 0, \quad (14)$$

$$u, \frac{\partial u}{\partial z}, v, \frac{\partial v}{\partial z} \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty, \quad t > 0. \quad (15)$$

Defining

$$F = u + iv, \quad (16)$$

Eqs. (12) and (13) can be combined as

$$\begin{aligned} \left(1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial F(z, t)}{\partial t} + \left(2i\Omega + \frac{\sigma B_0^2}{\rho} \right) \left(1 + \lambda \frac{\partial}{\partial t} \right) \\ \times F(z, t) + \frac{\nu \phi}{k} F(z, t) = \nu \frac{\partial^2 F(z, t)}{\partial z^2}, \quad z, t > 0, \end{aligned} \quad (17)$$

where ν is the kinematic viscosity. This equation, for Maxwell fluid, is one order higher in t than the corresponding equation for a Newtonian or second grade fluid. Consequently, in order to obtain unique solution for such a fluid, an additional initial condition is necessary. For a detail discussion about this problem and for some simple examples we refer the reader to [13,14]. Following Srivastava [15], we shall assume that the time derivative of velocity vanishes at time $t = 0$ [16,17]. As a result, the appropriate initial and boundary conditions are

$$\begin{aligned} F(0, t) = U_0, \quad t > 0, \\ F(z, 0) = \frac{\partial F(z, 0)}{\partial t} = 0, \quad z > 0, \end{aligned} \quad (18)$$

and

$$F(z, t), \frac{\partial F(z, t)}{\partial z} \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty, \quad t > 0. \quad (19)$$

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