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Coherence resonance in an excitable system with time delay

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Abstract

We study the noise activated dynamics of a model excitable system that consists of a subcritical Hopf oscillator with a time delayed nonlinear feedback. The coherence of the noise driven pulses of the system exhibits a novel double peaked structure as a function of the noise amplitude. The two peaks correspond to separate optimal noise levels for excitation of single spikes and multiple spikes (bursts) respectively. The relative magnitudes of these peaks are found to be a sensitive function of time delay. The physical significance of our results and its practical implications in various real life systems are discussed.

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Keywords: Coherence resonance; Excitable system; Time delayed feedback

1. Introduction

The constructive role of noise in the dynamics of complex systems is a subject of much current interest and activity. Important manifestations of such a behaviour are seen in basic phenomena like stochastic resonance (SR) [1–3], coherence resonance (CR) [4] or noise-induced synchronization of dynamical systems [5,6]. Coherence resonance (CR) which refers to the resonant response of a dynamical system to pure noise, is closely related to the phenomenon of stochastic resonance and is sometimes also known as autonomous stochastic resonance (ASR) [7]. The effect, first noticed by Sigeti and Horsthemke [8] in a general system at a saddle-node bifurcation, implies that a characteristic correlation time of the noise-excited oscillations has a maximum for a certain noise amplitude. This has been clearly demonstrated for the classic FitzHugh–Nagumo neuron model [4] and shown to have a deep connection to the excitable nature of the system. CR can have important consequences for neurophysiology or other complex systems where a significant degree of order can arise through interaction with a noisy environment. Past studies of CR have been mainly confined to simple systems whose noise-induced nonlinear outputs consist of impulsive excitations of a single kind e.g. spikes which have two characteristic time scales—a fast rise time and a longer decay time. This phenomenon has been found not only in various lab experiments, such as electronic circuits [9], laser systems [10,11], electrochemistry [12], or BZ reactions [13] but also in natural systems such as ice ages in climatology [14] or dynamos [15]. The behaviour of time-delayed bistable systems under the influence of noise has been studied in the past [16,17]. The impact of noise near different bifurcation states in the context of amplifying the stochastic resonance effects has been a subject matter of a number of previous studies [18–20].

As is well known, excitable systems especially in neuroscience, can also have a more complex response in terms of various time scales, such as short time spikes and bursts (multispiking) with different temporal signatures [21,22]. The nature of CR in the presence of different kinds of excitations (e.g.

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spikes and bursts) is not known and is one of the important objectives of the present study. In order to explore this question, we study the noise activated dynamics of a model excitable system that was recently presented in [23]. We find that the CR curve is able to discriminate between the two different kinds of excitations that the system possesses by displaying a bi-modal structure. Further, the relative predominance of the two peaks is a sensitive function of the time delay parameter.

2. The model

The model consists of a subcritical Hopf oscillator with a time delayed nonlinear feedback. By construction it contains the essential ingredients to reproduce most of the basic features of excitability that are displayed by standard neuronal models such as the Hodgkin–Huxley system [24] and in addition it provides a convenient means of studying the effect of time delay on the dynamical behavior. The basic mathematical form of the model is,

$$\dot{z}(t) = \left[i\left(\omega + b\left|z(t)\right|^2\right) + \left|z(t)\right|^2 - \left|z(t)\right|^4\right]z(t) - kz^2(t-\tau), \tag{1}$$

where $z = x + iy$ is a complex amplitude and the frequency of the oscillations is determined by ω and $b|z|^2$. The parameter b which is called the shear parameter, determines how the frequency depends on the amplitude of the oscillations. k is the magnitude of the feedback strength and τ is the time delay parameter. In the absence of the feedback term the oscillator is poised at the subcritical bifurcation point. The nonlinear feedback term, which provides the basic excitable behavior, is time delayed to account for finite propagation times of signals. The overall dynamical behaviour of the system is best captured in a two parameter bifurcation diagram (in k and τ space) which was obtained in [23] and is reproduced here as Fig. 1. The diagram delineates the different bifurcation branches as well as the regions of stable limit cycle, stable fixed point and bistability. When subjected to an external periodic signal and noise the system displays both spike trains as well as multi-spiking (bursty) behaviour [23]. For our present CR studies, we examine the temporal response of the system in the presence of only an external additive noisy stimulus, namely $f(t) = \sqrt{2D}\xi(t)$, where $\xi(t)$ is zero mean Gaussian white noise with intensity D. We confine ourselves to values of k that are above the critical value of $k_c = 0.42506$ and to τ values below the bi-stable region and choose different noise strengths D. The value of the shear parameter b has been chosen as -0.5. Its magnitude determines the location of the critical feedback strength k_c. However the nature of the bifurcation diagram (and the concomitant excitability) remains the same as long as $-1 < b \leqslant -0.5$.

3. Simulation results

For each noise intensity D we execute rather long simulations (100 datasets consisting of 200 000 time steps each with $\delta t = 0.05$) and collect a large number of interspike intervals (ISI) T. Using this data we determine the standard parameter for coherence resonance R, which is given by the ratio of the mean of the interspike intervals ($\langle T\rangle$) and its standard deviation (σ_T) [4,22,25,26],

$$R = \langle T\rangle/\sigma_T. \tag{2}$$

If the data consists of a completely random (Poissonian) set of spikes then R would have a value of unity, whereas a strictly periodic spiking train (maximum order) would make $R = \infty$. In general $R > 1$ indicates the presence of coherence. Our results for the model system are shown in Fig. 2 where the upper block shows the CR and the lower block depicts the average interspike interval (ISI) as functions of the noise intensity D. The solid curves are for $\tau = 0$ and the dashed curves correspond to finite time delay (in this case $\tau = 0.3$). We observe that unlike the standard CR results of a unimodal response curve [4] our present system produces a more complex response consisting of one well-expressed peak and, additionally, a broader peak in another region of noise intensities. The two-peaked structure is preserved in the presence of time delay with however one very

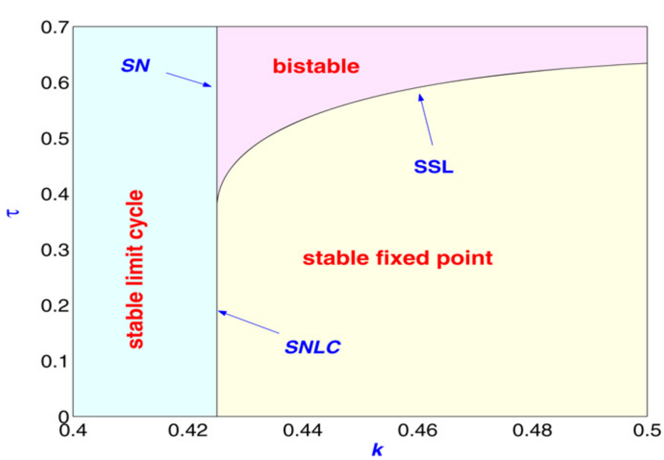

Fig. 1. (Color online.) Stability diagram in the parameter space of k and τ. Note the various bifurcation boundaries demarcating the different stability regions. (SNLC: Saddle-node on limit cycle bifurcation, SN: Saddle-node bifurcation, SSL: Saddle-separatrix loop bifurcation.)

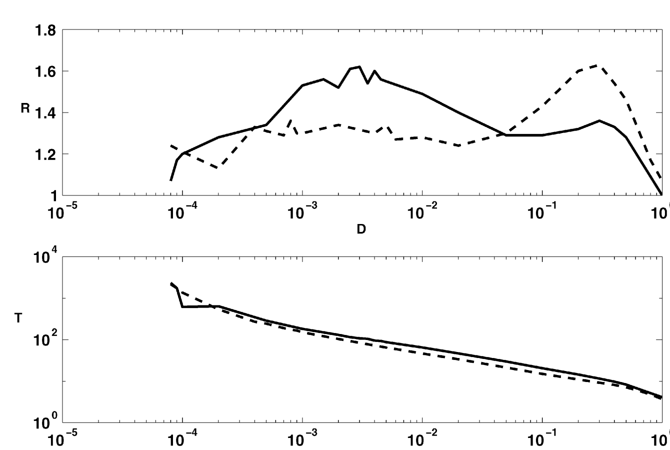

Fig. 2. The CR measure R and mean interspike interval $\langle T\rangle$ versus the noise intensity D. The solid curve is for $\tau = 0$ and the dashed curve is for $\tau = 0.30$. The feedback strength is $k = 0.426$. The point is close to SNLC bifurcation branch.