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The generation and circuit implementation of a new hyper-chaos based upon Lorenz system

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Abstract

This Letter presents a new hyper-chaotic system, which was obtained by adding a nonlinear quadratic controller to the second equation of the three-dimensional autonomous modified Lorenz chaotic system. The resulting hyper-chaotic system undergoes a change from hyper-chaos to limit cycle with some of its parameters changed. The phenomena were demonstrated by numerical simulations, bifurcation analysis and electronic circuit realization. The experiment results of the hyper-chaotic circuit were well agreed with the simulation results.

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1. Introduction

Hyper-chaos has been studied within many contexts such as Colpitts oscillator [1], nonlinear circuits [2], communication [3] and synchronization [4]. Hyper-chaotic Chua's circuit [5] and Rössler system [6] are two well-known examples. Due to its great potential in technological applications [7], the generation of hyper-chaos by circuits has become a focal research topic recently [2,8].

Hyper-chaos was first reported by Rössler in 1979. Since then, some other hyper-chaos has also been found [5,8,9]. Generating a hyper-chaotic attractor, in particular purposefully designing a hyper-chaotic system from an originally chaotic but non-hyper-chaotic system with some simple feedback control techniques, is a theoretically very attractive and yet technically quite challenging task. Li et al. designed a hyper-chaos through adding a state-feedback controller to the first control input to drive a unified chaotic system to generate hyper-chaos, and it was demonstrated by bifurcation analysis and an electronic circuit implementation [10,11]. Murali et al. presented the methodology of generating simple hyper-chaos circuits with a stable and an unstable oscillator, which was demonstrated by means of different simple hyperchaotic circuits with core RC sinusoidal oscillator and diode as the single nonlinear element [13].

This Letter presents a new hyper-chaotic system, which is generated by driving the Lorenz system [12] with a quadratic term controller. The generated hyper-chaotic system is not only demonstrated by numerical simulations but also verified with careful bifurcation analysis. Moreover, it is implemented via an electronic circuit and tested experimentally in laboratory, showing very good agreement with the simulation results.

The Letter is organized as follows. In Section 2, the hyper-chaos is introduced. In Section 3, several simulations are carried out to give a clear observation on the new chaotic attractor. In Section 4, some bifurcation analyses about the hyper-chaos are

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given. Section 5 introduces the implementation of the hyper-chaotic system via an electronic circuit. Finally, some concluding remarks and conclusions are given.

2. Generating hyper-chaos via a dynamical nonlinear controller

Consider the Lorenz chaotic system [12]

$$\begin{cases} \dot{x} = 10(y - x), \\ \dot{y} = 28x - xz + y, \\ \dot{z} = xy - \frac{8}{3}z. \end{cases} \tag{1}$$

The corresponding chaotic attractor is depicted in Fig. 1(a)–(b), which has a single positive Lyapunov exponent, $\lambda_1 = 1.069$, while the others are $\lambda_2 = 0$ and $\lambda_3 = -12.73$, respectively. More detailed complex dynamics of the Lorenz system can be seen in Ref. [12].

We know that, in order to obtain hyper-chaos, two important requisites are as follows:

(1) The minimal dimension of the phase space that embeds a hyper-chaotic attractor should be at least four, which requires the minimum number of coupled first-order autonomous ordinary differential equations to be four.
(2) The number of terms in the coupled equations giving rise to instability should be at least two, of which at least one should have a nonlinear function [6].

By introducing a simple quadratic dynamic feedback control term w to the second equation of system (1), the following four-dimensional dynamic system is obtained

$$\begin{cases} \dot{x} = 10(y - x), \\ \dot{y} = 28x + y - xz - w, \\ \dot{z} = xy - \frac{8}{3}z, \\ \dot{w} = kyz, \end{cases} \tag{2}$$

where k is a constant, determining the chaotic attractor and bifurcations of system (2). Obviously, the chaotic system (2) is a four-dimensional dynamical system, which has four Lyapunov exponents. This may lead to a hyper-chaotic system. Obviously, the Jacobian matrix of (2) is

$$\begin{pmatrix} -10 & 10 & 0 & 0 \\ 28 - z & 1 & -x & -1 \\ y & x & -\frac{8}{3} & 0 \\ 0 & kz & ky & 0 \end{pmatrix}. \tag{3}$$

Its eigenvalues at origin are $\lambda_1 = -2.667$, $\lambda_2 = 13.11$, $\lambda_3 = -22.11$ and $\lambda_4 = 0$.

3. Observation of new chaotic attractors

In system (2), when parameter k varies, several simulations have been carried out, the outcome of chaotic attractors and period-doubling bifurcations are summarized as follows:

(1) When $k = 0.1$, the hyper-chaos strange attractors are shown in Fig. 2(a) and (b), the attractor is still bounded at this time.
(2) When $k = 0.2$, the corresponding strange attractors are shown in Fig. 2(c) and (d).
(3) When $k = 0.22$, system enters into periodic orbits, the portraits of states are shown in Fig. 2(e) and (f).
(4) When $k = 0.26$, the periodic states are shown in Fig. 2(g) and (h). Obviously, they are different from those when $k = 0.22$.
(5) When $k = 0.30$, the periodic states are shown in Fig. 2(i) and (j). Obviously, they are also different from those in the above situations.
(6) When $k = 0.4$, the corresponding chaos strange attractors are shown in Fig. 2(k) and (l).
(7) When $k = 0.6$, the periodic states are shown in Fig. 2(m) and (n).

It can be seen from the simulation results that the system undergoes hyper-chaos, chaos, and some different periodic orbits when the parameter k varies.

4. Bifurcation analysis

There does not seem to be any systematic methodology for purposefully designing a hyper-chaotic system to date. Therefore, the following investigation relies on a combination of

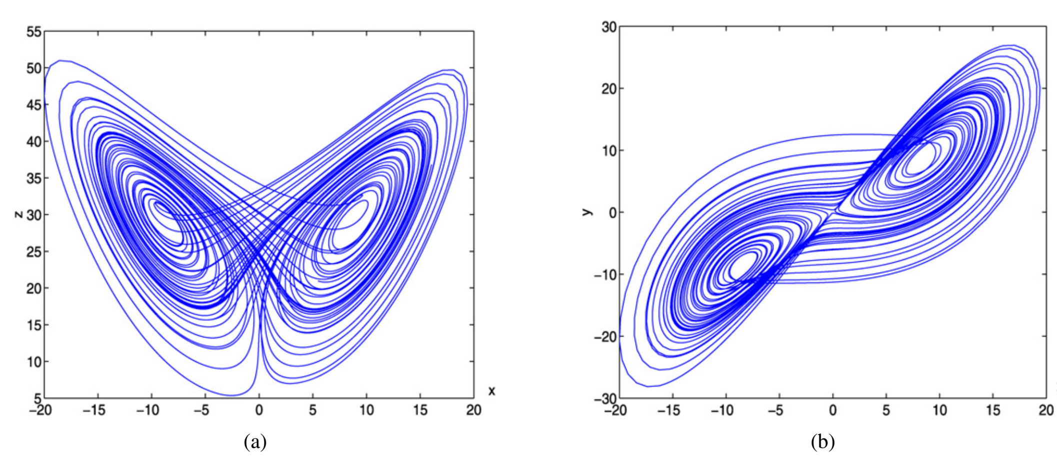

Fig. 1. Phase portraits of chaotic system (1): (a) x–z plane, (b) x–y plane.