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Complete flexural vibration band gaps in membrane-like lattice structures

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Abstract

The propagation of flexural vibration in the periodical membrane-like lattice structure is studied. The band structure calculated with the plane wave expansion method indicates the existence of complete gaps. The frequency response function of a finite periodic structure is simulated with finite element method. Frequency ranges with vibration attenuation are in good agreement with the gaps found in the band structure. Much larger attenuations are found in the complete gaps comparing to those directional ones. The existence of complete flexural vibration gaps in such a lattice structure provides a new idea for vibration control of thin plates.
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1. Introduction

The elastic wave propagation in periodic structures was researched for years [1–5]. The vibration response of periodic structures has been applied primarily to analysis pass band and stop band. But most of these works were dealing with one-dimensional structures with transfer matrix method [3–5].

In the last decade, the propagation of elastic or acoustic waves in periodic composites, so-called phononic crystals (PCs), has received much attention [6–10]. This is of interest for applications such as frequency filters, vibrationless environments for high-precision mechanical systems or design of new transducers. By using the calculation method of PCs, the vibration propagation property of two-dimensional and three-dimensional periodic structures can be dealt with conveniently.

Vibration band gaps in PCs have also been found experimentally and theoretically [11–16]. In Ref. [15], the authors demonstrated that for many mechanical lattice structures there are intervals of frequencies within which no propagating elastic waves exist. They also presented a method to determine such band gaps and they can show how lattices can be constructed that have band gaps around prescribed frequencies. For example, they considered a membrane-like periodic structure and provided the analytical dispersion equation. But it is not applicable to describe its flexural vibration property.

The flexural waves propagating in composite plates consisting of cylindrical inclusions periodically placed in a host material had been studied theoretically [8]. The flexural waves velocity is proportional to the in plane wave vector and not a constant as in an infinite medium, the difference makes the appearance of a full gap more difficult [8]. So they only found the directional flexural vibration band gaps in the thin plates.

In this Letter, we investigate the flexural vibration band gaps in a lattice structure. The band structure of the flexural wave is calculated by the plane wave expansion (PWE) method that is proved reliable. The complete band gaps of flexural vibration can

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exist in such lattice structures. Finally, the frequency response function (FRF) is calculated by the finite element (FE) method to validate our theory.

2. PWE method

Fig. 1 shows the sketch of a square lattice structure with three kinds of material. The black part is material A and the white part is material B, the rest part in one cell is vacuum. The periodicity of the square lattice is $a = 2(l + d)$. The thickness of the structure is h. If $h \ll a$, we can treat the structure as a thin plate.

It is well known that the conventional PWE method is not suitable for solid/liquid or solid/air system. Liquid or air does not support the propagation of transverse (or both the transverse and longitudinal) waves. Therefore the PWE method fails by producing unphysical flat frequency bands. But it is applicable for solid/vacuum systems with some manipulation to the calculation of the dispersion relations [16,17]. So PWE method can be used to calculate the band structure of the lattice structure illustrated in Fig. 1.

The wave equations of flexural wave for thin plates of thickness h is known to be [8]

$$-\rho h\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left[D\left(\frac{\partial^2 w}{\partial x^2} + \sigma\frac{\partial^2 w}{\partial y^2}\right)\right] + 2\frac{\partial^2}{\partial x\partial y}\left[D(1-\sigma)\frac{\partial^2 w}{\partial x\partial y}\right] + \frac{\partial^2}{\partial y^2}\left[D\left(\frac{\partial^2 w}{\partial y^2} + \sigma\frac{\partial^2 w}{\partial x^2}\right)\right], \tag{1}$$

where w is the flexural displacement in the z direction, $D = Eh^3/12(1-\sigma^2)$ is the flexural rigidity.

Eq. (1) can be rewritten as

$$-\alpha\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial x^2} + \beta\frac{\partial^2 w}{\partial y^2}\right) + 2\frac{\partial^2}{\partial x\partial y}\left(\gamma\frac{\partial^2 w}{\partial x\partial y}\right) + \frac{\partial^2}{\partial y^2}\left(D\frac{\partial^2 w}{\partial y^2} + \beta\frac{\partial^2 w}{\partial x^2}\right), \tag{2}$$

where $\alpha = \rho h$, $\beta = D\sigma$, $\gamma = D(1-\sigma)$.

Following the plane wave expansion methodology, Eq. (2) can be derived in the form of a standard eigenvalue problem at hand:

$$\begin{aligned}
&\omega^2 \sum_{G'} \alpha(\mathbf{G}'' - \mathbf{G}') w_{\mathbf{k}+\mathbf{G}'}(\mathbf{G}') \\
&\quad= \sum_{G'} (\mathbf{k}+\mathbf{G}')_x^2 (\mathbf{k}+\mathbf{G}'')_x^2 D(\mathbf{G}''-\mathbf{G}') w_{\mathbf{k}+\mathbf{G}'}(\mathbf{G}') + \sum_{G'} (\mathbf{k}+\mathbf{G}')_y^2 (\mathbf{k}+\mathbf{G}'')_x^2 \beta(\mathbf{G}''-\mathbf{G}') w_{\mathbf{k}+\mathbf{G}'}(\mathbf{G}') \\
&\qquad+ 2\sum_{G'} (\mathbf{k}+\mathbf{G}')_x (\mathbf{k}+\mathbf{G}')_y (\mathbf{k}+\mathbf{G}'')_x (\mathbf{k}+\mathbf{G}'')_y \gamma(\mathbf{G}''-\mathbf{G}') w_{\mathbf{k}+\mathbf{G}'}(\mathbf{G}') \\
&\qquad+ \sum_{G'} (\mathbf{k}+\mathbf{G}')_y^2 (\mathbf{k}+\mathbf{G}'')_y^2 D(\mathbf{G}''-\mathbf{G}') w_{\mathbf{k}+\mathbf{G}'}(\mathbf{G}') + \sum_{G'} (\mathbf{k}+\mathbf{G}')_x^2 (\mathbf{k}+\mathbf{G}'')_y^2 \beta(\mathbf{G}''-\mathbf{G}') w_{\mathbf{k}+\mathbf{G}'}(\mathbf{G}'),
\end{aligned} \tag{3}$$

where $\mathbf{k}$ is Bloch wave vector restricted within the first Brillouin zone, and $\mathbf{G}''$ and $\mathbf{G}'$ are reciprocal vectors, and D, α, β and γ are periodic function of $\mathbf{r} = (x, y)$. $\alpha(\mathbf{G})$, $\beta(\mathbf{G})$, $\gamma(\mathbf{G})$ and $D(\mathbf{G})$ are the Fourier coefficient. And the Fourier coefficient can be easily shown as follows:

$$f(\mathbf{G}) = \begin{cases} F_A f_A + F_B f_B, & \mathbf{G} = \mathbf{0}, \\ f_A P_1(\mathbf{G}) + f_B P_2(\mathbf{G}), & \mathbf{G} \neq \mathbf{0}, \end{cases} \tag{4}$$

where f can be one of α, β, γ and D, A and B label material A and material B, F_A and F_B are filling ratio of material A and material B respectively, i.e., $F_A = 4(dl + d^2)/a^2$ and $F_B = 4dl/a^2$. Since there are no stresses in a vacuum, we set parameters D, β and γ to zeros. As a consequence, we set $\alpha = 0$ basing on Eq. (2). This technique had been adopted for the PWE method to calculate the band structure of the solid/vacuum system [17]. $P_1(\mathbf{G})$ and $P_2(\mathbf{G})$ are structure functions. It can be shown that the

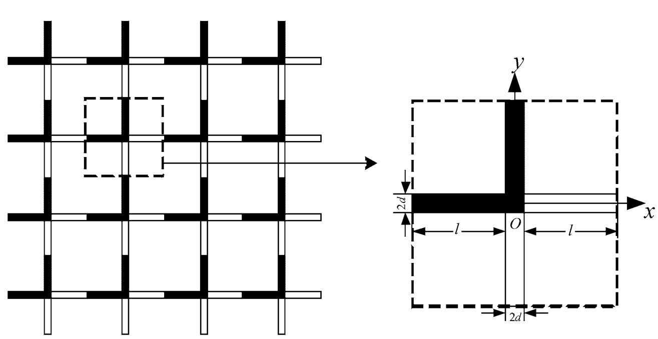

Fig. 1. Square lattice structure with three kinds of material. The black part is material A and the white part is material B, the rest part in one cell is vacuum.

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