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Design of bursting in a two-dimensional discrete-time neuron model

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Abstract

Alternating a quiescent state and a spiking one in a neuron is called *bursting*, which is one of important neuron activities. In this Letter, we will propose a simple design method for a bursting neuron model with a specified period and duty ratio of the bursting based on bifurcation theory. The neuron model has been proposed by the author et al. based on Aihara's chaotic neuron model. Mechanism of generating the bursting in the neuron model is a quasi-periodic oscillation with respect to internal states, which is caused from a Hopf bifurcation for a pair of two-periodic points. We also show an example derived by the proposed design method.

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1. Introduction

When neuron activity alternates between a quiescent state and repetitive spiking (a spiking state), the neuron activity is said to be *bursting*. The bursting oscillations can be observed in various biological neurons [1]. Moreover, it is believed that the bursting plays crucial roles: For example, they facilitate secretion of hormones [2,3] and contribute effective muscle contraction [4].

On the other hand, many bursting neuron models have been proposed. In continuous-time neuron models, Izhikevich has clarified mechanisms of generation of the bursting from bifurcation theoretical point of view [5]. In discrete-time neuron models, Izhikevich and Hoppensteadt have classified bursting mappings [6].

In this Letter, we pay attention to discrete-time bursting neuron models. Since it has been reported that bursting can be generated by interactions between fast and slow subsystems, some discrete-time bursting neuron models with both subsystems have been proposed. For example, Rulkov has proposed a two-dimensional bursting neuron model containing fast and slow variables [7]. Moreover, Shilnikov and Rulkov have reported that the model generates chaos [8]. However, the model does not have continuous dynamics. Therefore, Rulkov has also proposed a two-dimensional bursting neuron model [9]. The model has continuous dynamics and contains both fast and slow variables.

However, some bursting neuron models without such subsystems have been proposed. For example, Cazelles et al. have proposed a one-dimensional bursting neuron model [10], which has discontinuous dynamics. We have also proposed a high-dimensional bursting neuron model [11]. The model does not contain fast and slow subsystems and is derived by extending Aihara's chaotic neuron model [12]. Since Aihara's chaotic neuron has one-dimensional dynamics, it is difficult to observe bursting oscillations. However, Kitajima et al. have observed and analyzed bursting in a neural network with ring structure [13], which is constructed by Aihara's neurons.

As we described above, bursting can be observed in biological neurons [1]. It is important for neuron models to generate such a bursting behavior because the neuron models are proposed in order to simulate neuron activity. However, it is shown that there are various kinds of shapes of bursting [1]. Therefore, we focus on a period and duty ratio of bursting. The period is sum of duration of the quiescent and spiking states. The duty

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ratio is defined by proportion of the period to duration of the spiking state. We will propose a simple design method of our neuron model [11] for the bursting with specified period and duty ratio in this Letter. The proposed method derives parameters and an initial conditions.

This Letter is organized as follows: In Section 2, a considered bursting neuron model in this Letter will be introduced, and mechanism of bursting in the model will be explained. Then, in Section 3, we will propose a design method of the bursting. In Section 4, we will show an example by applying the proposed method. Finally, we will conclude this Letter in Section 5.

2. Preliminaries

In this section, we firstly describe a two-dimensional discrete-time neuron model proposed by the authors [11]. Secondly, we explain mechanism of generation bursting in the model. Since the mechanism is closely related to a Hopf bifurcation for a pair of two-periodic points, we thirdly derive a necessary condition for the Hopf bifurcation. Finally, we define specifications (a period and duty ratio) of bursting.

2.1. A neuron model

We consider a two-dimensional discrete-time neuron model [11] given by

$$\begin{cases} y_1(t+1) = k_1 y_1(t) + k_2 y_2(t) - \alpha f(y_1(t)) + c, \\ y_2(t+1) = y_1(t), \end{cases} \tag{1}$$

where t is a discrete time, $(y_1(t), y_2(t))$ is an internal state, and k_1, k_2, α, and c are parameters. The output from the neuron is determined by

$$x(t) = f\bigl(y_1(t)\bigr) = \frac{1}{1 + \exp(-y_1(t)/\varepsilon)}, \tag{2}$$

where ε is a steepness parameter.

2.2. Mechanism of the bursting

In this subsection, we review mechanism of generating bursting oscillations in the model. Please see [11] for detail.

There are some fixed points and some pairs of two-periodic points in the model. Fig. 1 shows a phase portrait when a Hopf bifurcation for two pairs of the two-periodic points occurs [11]. The parameters are

$$k_1 = 2.0, \quad k_2 = 1.0, \quad \alpha = 1.0, \quad c = 0.5, \quad \varepsilon = 0.1. \tag{3}$$

In Fig. 1, there are three fixed points denoted by O (the origin) and $P^{\pm}$. There also exist three pairs of two-periodic points denoted by $Q_i^{\pm}$ $(i = 1, 2, 3)$. In this case, a Hopf bifurcation for $Q_2^{\pm}$ and $Q_3^{\pm}$ occurs at the same time while all the fixed points and the other pair of the two-periodic point $Q_1^{\pm}$ are saddle points.

Since a Hopf bifurcation for them occurs, there exists a pair of non-isolated invariant closed curves surrounding each two-periodic point. The curves surrounding $Q_3^{\pm}$ are important for

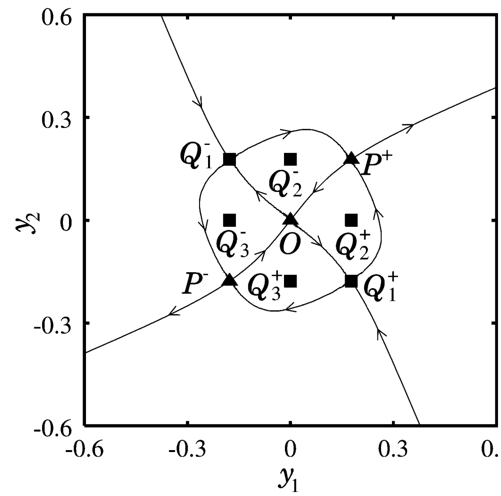

Fig. 1. A phase portrait.

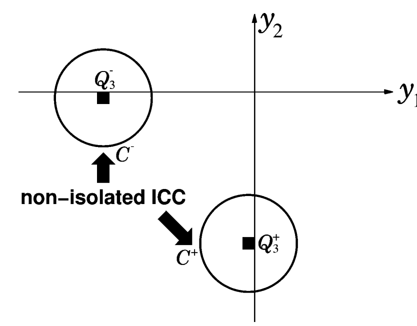

Fig. 2. Bursting and non-isolated invariant closed curves.

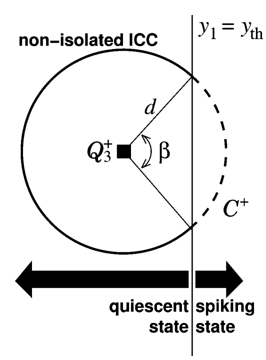

Fig. 3. Bursting on the phase plane, where y_{th} is a threshold.

generation of bursting in this Letter. Fig. 2 shows $Q_3^{\pm}$ and one pair of the non-isolated invariant closed curves denoted $C^{\pm}$.

The state's behavior visits the invariant closed curves $C^{\pm}$ alternately because they result from the Hopf bifurcation for the two-periodic points $Q_3^{\pm}$. Fig. 3 is enlargement of Fig. 2 around the two-periodic point Q_3^{+} and y_{th} is a threshold for firing or non-firing with respect to an internal state y_1, i.e., if $y_1(t) > y_{th}$ (respectively, $y_1(t) < y_{th}$), the neuron is firing (respectively, non-firing) at the time t.

If a state is on the dashed arc on C^{+} in Fig. 3, the neuron is firing at the time and non-firing at the next time because the state is on the other non-isolated invariant closed curve. Consequently, the neuron is in the spiking state. On the other hand, if a