

Global exponential stability of cellular neural networks with continuously distributed delays and impulses [☆]

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Abstract

In this Letter cellular neural networks with continuously distributed delays and impulses are considered. Sufficient conditions for the existence and global exponential stability of a unique equilibrium point are established by using the fixed point theorem and differential inequality techniques. The results of this Letter are new and they complement previously known results.

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1. Introduction

Consider the cellular neural networks (CNNs) with continuously distributed delays and impulses as follows

$$\begin{cases} x'_i(t) = -a_i x_i(t) + \sum_{j=1}^n p_{ij} f_j(x_j(t)) + \sum_{j=1}^n q_{ij} \int_0^\infty k_{ij}(s) g_j(x_j(t-s)) ds + c_i, & t > 0, t \neq t_k, \\ \Delta x_i(t_k) = I_k(x_i(t_k)), & i = 1, 2, \dots, n, k = 1, 2, \dots, \end{cases} \quad (1.1)$$

where n corresponds to the number of neurons. For $i, j = 1, 2, \dots, n$, $x_i(t)$ is the activations of the i th neuron; a_i is a positive constant and represents the rate with which the i th neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; c_i is a constants and denotes the external inputs; p_{ij} and q_{ij} are constants and denote the connection weights at the time t , $k_{ij}(t)$ is the delayed feedback, and f_j and g_j are signal transmission functions; $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$ is the impulse at moments t_k and $t_1 < t_2 < \dots$ is a strictly increasing sequences such that $\lim_{k \rightarrow +\infty} t_k = +\infty$.

In the following we assume that conditions (C₁) and (C₂) hold.

(C₁) The signal transmission functions $f_j, g_j, j = 1, 2, \dots, n$, are Lipschitz continuous on R with Lipschitz constants L_j^f and L_j^g , that is,

$$|f_j(x) - f_j(y)| \leq L_j^f |x - y|, \quad |g_j(x) - g_j(y)| \leq L_j^g |x - y|, \quad \forall x, y \in R.$$

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(C₂) For $i, j = 1, 2, \dots, n$, are continuous functions, $\int_0^\infty |k_{ij}(s)| ds$ is existent, and there exist nonnegative constants p_{ij}^+ , q_{ij}^+ and k_{ij}^+ such that

$$p_{ij}^+ = |p_{ij}|, \quad q_{ij}^+ = |q_{ij}|, \quad \int_0^\infty |k_{ij}(s)| ds \leq k_{ij}^+.$$

It is well known that the CNNs have been successfully applied to signal and image processing, pattern recognition and optimization. Hence, they have been the object of intensive analysis by numerous authors in recent years. In particular, there have been extensive results on the problem of the stability and other dynamical behaviors of impulsive CNNs with constant time delays and time-varying delays in the literature. We refer the reader to [1–12] and the references cited therein. However, there exist few results on the dynamical behaviors of CNNs with distributed delays and impulses.

In this Letter, by using the fixed point theorem and differential inequality techniques, we will give some new sufficient conditions for the existence and exponential stability of the equilibrium point for system (1.1). The results of this Letter are new and they complement previously known results.

The initial conditions associated with (1.1) are assumed to be of the form

$$x_i(s) = \phi_i(s), \quad s \in (-\infty, 0], \quad i = 1, 2, \dots, n, \quad (1.2)$$

where $\phi_i(\cdot)$ denotes real-valued bounded continuous function defined on $(-\infty, 0]$.

As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $t \mapsto (x_1(t), x_2(t), \dots, x_n(t))^T$, we assume that $(x_1(t), x_2(t), \dots, x_n(t))^T \equiv (x_1(t-0), x_2(t-0), \dots, x_n(t-0))^T$. It is clearly that, in general, the derivatives $x'_i(t_k)$ does not exist. On the other hand, according to system (1.1), there exist the limits $x'_i(t_k \mp 0)$. In view of the above convention, we assume that $x'_i(t_k) \equiv x'_i(t_k - 0)$.

For convenience, we introduce some notations. We will use $x = (x_1, x_2, \dots, x_n)^T \in R^n$ to denote a column vector, in which the symbol $(^T)$ denotes the transpose of a vector. For matrix $D = (d_{ij})_{n \times n}$, D^T denotes the transpose of D , and E_n denotes the identity matrix of size n . A matrix or vector $D \geq 0$ means that all entries of D are greater than or equal to zero. $D > 0$ can be defined similarly. For matrices or vectors D and E , $D \geq E$ (respectively, $D > E$) means that $D - E \geq 0$ (respectively, $D - E > 0$).

An equilibrium point of (1.1) is a constant vector $(x_1^*, x_2^*, \dots, x_n^*)^T \in R^n$ which satisfies the system

$$a_i x_i^* = \sum_{j=1}^n p_{ij} f_j(x_j^*) + \sum_{j=1}^n q_{ij} \int_0^\infty k_{ij}(s) g_j(x_j^*) ds + c_i, \quad i = 1, 2, \dots, n, \quad (1.3)$$

when the impulsive jump $I_k(\cdot)$ as assume to satisfy $I_k(x_i^*) = 0$, $i = 1, 2, \dots, n$, $k \in Z^+$, where Z^+ denotes the set of all positive integers. In what follows, we use following norm of R^n :

$$\|w\| = \max_{1 \leq i \leq n} |w_i|, \quad \text{for } w = (w_1, w_2, \dots, w_n)^T \in R^n.$$

Definition 1. Let $Z^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ be an equilibrium point of system (1.1). If there exist constants $\lambda > 0$ and $M_\varphi > 1$ such that for every solution $Z(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ of system (1.1) with any initial value $\varphi = (\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t))^T$,

$$|x_i(t) - x_i^*| \leq M_\varphi \|\varphi - Z^*\|_1 e^{-\lambda t}, \quad i = 1, 2, \dots, n, \quad \forall t > 0,$$

where $\|\varphi - Z^*\|_1 = \max_{1 \leq i \leq n} \sup_{-\infty \leq t \leq 0} |\varphi_i(t) - x_i^*|$. Then Z^* is said to be globally exponentially stable.

Definition 2. A real $n \times n$ matrix $A = (a_{ij})$ is said to be an M -matrix if $a_{ij} \leq 0$, $i, j = 1, 2, \dots, n$, $i \neq j$, and $A^{-1} \geq 0$.

Lemma 1.1 [13,14]. Let $A = (a_{ij})_{n \times n}$ with $a_{ij} \leq 0$, $i, j = 1, 2, \dots, n$, $i \neq j$. Then the following statements are equivalent:

- (1) A is an M -matrix.
- (2) There exists a vector $\eta = (\eta_1, \eta_2, \dots, \eta_n) > (0, 0, \dots, 0)$ such that $\eta A > 0$.
- (3) There exists a vector $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T > (0, 0, \dots, 0)^T$ such that $A \xi > 0$.

Lemma 1.2 [13,14]. Let $A \geq 0$ be an $n \times n$ matrix and $\rho(A) < 1$, then $(E_n - A)^{-1} \geq 0$, where $\rho(A)$ denotes the spectral radius of A .

The remaining part of this Letter is organized as follows. In Section 2, we shall derive new sufficient conditions for checking the existence and uniqueness of the equilibrium point. In Section 3, we present some new sufficient conditions for the exponential

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