

Exponential stability of Cohen–Grossberg neural networks with a general class of activation functions [☆]

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Abstract

In this Letter, the dynamics of Cohen–Grossberg neural networks model are investigated. The activation functions are only assumed to be Lipschitz continuous, which provide a much wider application domain for neural networks than the previous results. By means of the extended nonlinear measure approach, new and relaxed sufficient conditions for the existence, uniqueness and global exponential stability of equilibrium of the neural networks are obtained. Moreover, an estimate for the exponential convergence rate of the neural networks is precisely characterized. Our results improve those existing ones.

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1. Introduction

In this Letter, we consider the Cohen–Grossberg neural networks model, which is described by the following differential equations [1]

$$\frac{du_i(t)}{dt} = -a_i(u_i(t)) \left[b_i(u_i(t)) - \sum_{j=1}^n w_{ij} f_j(u_j(t)) + I_i \right], \quad i = 1, 2, \dots, n, \quad (1)$$

where $n \geq 2$ is the number of neurons in the networks, $u_i(t)$ denotes the neuron state vector, a_i denotes an amplification function, b_i denotes a self-signal function, $W = (w_{ij})_{n \times n}$ is the connection matrix, f_i denotes an activation function, and I_i represents the constant external input.

Model (1) includes a large number of models from neurobiology and population biology [2]. In particular, it includes as a special case the popular Hopfield neural networks [3]. Moreover, Cohen–Grossberg neural networks have potential applications in many areas such as associative memory and optimization. Therefore, the investigation on Cohen–Grossberg neural networks is of fundamental theoretical and practical significance.

As is well known, stability of neural networks is fundamental for the designs and applications of neural networks [4–16]. In this Letter, we are devoted to the exponential stability analysis of (1). We only make the following assumptions:

(H₁) Each a_i is continuous and $0 < \hat{\alpha}_i \leq a_i(r) \leq \acute{\alpha}_i$ for any $r \in \mathbb{R}$.

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- (H₂) Each b_i is continuous, and there exists a constant $\lambda_i > 0$ such that $(r_1 - r_2)[b_i(r_1) - b_i(r_2)] \geq \lambda_i(r_1 - r_2)^2$ for any $r_1, r_2 \in \mathbb{R}$.
 (H₃) Each f_i is Lipschitz continuous. Denote m_i the minimal Lipschitz constant of f_i , i.e., $m_i = \sup_{r_1, r_2 \in \mathbb{R}, r_1 \neq r_2} \frac{|f_i(r_1) - f_i(r_2)|}{|r_1 - r_2|}$.

Recently, Wang et al. [12], Liao et al. [13] and Lu et al. [14], respectively, analyzed the global exponential stability of (1), but they additionally required that each f_i be bounded or monotonically increasing. However, in this letter, not only do we abandon the boundedness condition of f_i , but also we remove the differentiability and monotonicity restriction from f_i , although this will lead to more difficulty in stability analysis. In addition, we do not impose any restriction such as symmetry on the connection matrix.

To the best of the authors' knowledge, the most popular approach to stability analysis of neural networks is based on Lyapunov direct method (see, for example, [4,7,8,12–16] and references therein). However, no general rule can guide how a Lyapunov function should be constructed for a given system, and thus the construction of a proper Lyapunov function is usually rather difficult. This Letter aims to analyze the stability of Cohen–Grossberg neural networks (1) by means of the extended nonlinear measure approach, and derive a series of new relaxed sufficient conditions for the exponential stability of (1).

2. Preliminaries

Let $\mathbb{R}^n$ denote the n -dimensional real vector space endowed with vector norm $\|\cdot\|$. Let $\langle \cdot, \cdot \rangle$ denote the inner product of vectors in $\mathbb{R}^n$ and $\text{sign}(x) = (\text{sign}(x_1), \text{sign}(x_2), \dots, \text{sign}(x_n))^T$ denote the sign vector of $x \in \mathbb{R}^n$, where $\text{sign}(r)$ represents the sign function of $r \in \mathbb{R}$. The l^1 -norm and l^2 -norm of x are defined by $\|x\|_1 = \sum_{i=1}^n |x_i|$ and $\|x\|_2 = (\sum_{i=1}^n x_i^2)^{1/2}$ for any $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$.

Let Ω be an open subset of $\mathbb{R}^n$. Consider the following system

$$\frac{dx(t)}{dt} = F(x(t)), \quad t \geq t_0, \quad (2)$$

where F is a nonlinear operator from Ω to $\mathbb{R}^n$ and $x(t) \in \Omega$.

Definition 1. Suppose that x^* is an equilibrium point of system (2). System (2) is said to be globally exponentially stable, if there exist constants $c \geq 1$ and $\sigma > 0$ such that $\|x(t) - x^*\| \leq ce^{-\sigma(t-t_0)}\|x_0 - x^*\|$ ($t \geq t_0$), where $x(t)$ is any solution of (2) initiated from $x_0 = x(t_0) \in \mathbb{R}^n$.

Definition 2 [11]. Suppose that Ω is an open subset of $\mathbb{R}^n$, F is an operator from Ω to $\mathbb{R}^n$, and x^0 is any fixed point in Ω . The constant

$$M_\Omega(F) = \sup_{x, y \in \Omega, x \neq y} \frac{\langle F(x) - F(y), \text{sign}(x - y) \rangle}{\|x - y\|_1} \quad (3)$$

is called the nonlinear measure of F on Ω . The constant

$$M_\Omega(F, x^0) = \sup_{x \in \Omega, x \neq x^0} \frac{\langle F(x) - F(x^0), \text{sign}(x - x^0) \rangle}{\|x - x^0\|_1} \quad (4)$$

is called the relative nonlinear measure of F at x^0 .

Inspired by Definition 2, we introduce the following extended nonlinear measure concepts of F in the sense of the l^2 -norm, namely:

Definition 3. Denote Ω , F and x^0 the same as in Definition 2. The constant

$$m_\Omega(F) = \sup_{x, y \in \Omega, x \neq y} \frac{\langle F(x) - F(y), x - y \rangle}{\|x - y\|_2^2} \quad (5)$$

is called the nonlinear measure of F on Ω . The constant

$$m_\Omega(F, x^0) = \sup_{x \in \Omega, x \neq x^0} \frac{\langle F(x) - F(x^0), x - x^0 \rangle}{\|x - x^0\|_2^2} \quad (6)$$

is called the relative nonlinear measure of F at x^0 .

It is clear that $m_\Omega(F, x^0) \leq m_\Omega(F)$. Now we will show that, the above nonlinear measures induced by the l^2 -norm have the following important properties analogous to Lemma 1 and Theorem 2 in [11].

Lemma 1. If $m_\Omega(F) < 0$, then F is a one-to-one mapping on Ω . If in addition $\Omega = \mathbb{R}^n$, then F is a homeomorphism of $\mathbb{R}^n$.

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