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Effective ac dielectric response of graded cylindrical composites

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Abstract

Under an external alternating current (ac) field, the effective ac dielectric response of graded composites consisting of the graded cylindrical inclusion having complex permittivity profiles has been investigated theoretically. A model that the dielectric function is assumed to be a constant while the conductivity has a power-law dependence on the radial variable r, namely $\varepsilon_i(r) = A + cr^k/i\omega$, is studied and the local analytical potentials of the inclusion and the host regions are derived in terms of hyper-geometric function. In the dilute limit, the effective ac dielectric response is predicted. Meanwhile, we have given the exact proof of the differential effective dipole approximation (DEDA) method, which is suitable to arbitrary graded profiles. Furthermore, we have given the analytical potentials and the effective ac dielectric responses of coated graded cylindrical composites for two cases, case (a) graded core and case (b) graded coated layer, having the graded dielectric profiles, respectively.
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1. Introduction

Graded composites have attracted much attention because their effective properties have advantages over traditional homogeneous composite materials [1–3]. The properties of graded composites are of wide applications to the engineering of functionally graded materials, such as the electricity, thermodynamics, mechanics and so on [3–6]. In laboratory, graded composites can be designed by varying their compositions and values of their physical properties along the radial direction of the cylindrical or spherical inclusions for the specific needs of engineering. For the purpose, it is necessary to predict the effective properties of the graded composite so that one could improve the design of engineering. In fact, the effective response of graded composites is not only related to the intrinsic constituent relations, microstructures, shapes and volume fraction of inclusions but also the properties of the materials and the external field.

For an external dc electric field, many of authors were devoted to investigate the effective response of graded composites. For example, Gu and Yu [7,8] derived the effective response of graded cylindrical composites having the power-law, linear and exponential electrical conductive profiles of graded cylindrical inclusions. Wei et al. [9,10] considered the graded cylindrical composites having the combinations of the general power-law and the exponential dielectric profiles. Dong et al. [11,12] and Huang et al. [13] discussed the effective response of graded composites having the linear and nonlinear constituent relations using the differential effective dipole approximation (DEDA). However, most of above works are only suitable to predict the real response of graded composites. In Nature, there are many of realistic materials having the complex dielectric permittivity. Therefore, under an external

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ac electric field, it is of practical significance to study the effective response of the graded composites having complex dielectric graded profiles. The local potentials induced by an external ac field give rise to some important physical properties of graded composites [14], such as the optical properties. However, the lack of analytic solutions of the local electric field has hindered the accurate analysis of physical properties of graded composites having complex graded profiles. Hence, in this Letter, we shall derive the analytical potentials of a theoretical model of the graded cylindrical composites under an external ac field.

We consider a graded cylindrical composite having a complex power-law dielectric profile $\varepsilon_i(r) = A + cr^k/i\omega$, where r is the radial variable of cylindrical inclusion, and A, c and k are constants and ω is the angular frequencies of external ac electric field. The exact solutions of the local potentials and the effective ac response are derived. Meanwhile, the effective ac responses of coated graded cylindrical composites are investigated for two cases, (a) the graded core and (b) graded coated layer. In Appendix A, we have given the exact proof of DEDA method.

2. Local potential in a graded cylindrical inclusion

For linear composites, the Maxwell equations read

$$\nabla \cdot \vec{D} = \rho, \tag{1}$$

$$\nabla \times \vec{E} = 0. \tag{2}$$

In Eq. (1), the free charge density ρ is related to the electrical conduction current density $\vec{J} = \sigma \vec{E}$, where σ is the electrical conductivity. By invoking the equation of continuity $\nabla \cdot \vec{J} + \partial\rho/\partial t = 0$ or $\nabla \cdot \vec{J} = -i\omega\rho$ due to the sinusoidal variation of the physical quantities, where $\omega = 2\pi f$ is the angular frequency of applied electrical field, we arrive at, from Eq. (1), $\nabla \cdot (\frac{\sigma(r)}{-i\omega}\vec{E}) = \rho = \nabla \cdot \varepsilon(r)\vec{E}$. From Eq. (2), $\vec{E}$ can be expressed as the gradient of a scalar potential Φ, $\vec{E} = -\nabla\Phi$, yielding a partial differential equation

$$\nabla \cdot \left[\tilde{\varepsilon}(r)\nabla\Phi\right] = 0, \tag{3}$$

where $\tilde{\varepsilon}(r) = \varepsilon(r) + \sigma(r)/i\omega$ is a complex permittivity of the graded inclusion. Therefore, from Eq. (3), the constitutive relation between the electric field and the electric displacement of graded composites having complex dielectric response can be rewritten as the form $\vec{D} = \tilde{\varepsilon}(r)\vec{E}$, and the governing equations are $\nabla \cdot \vec{D} = 0$ and $\nabla \times \vec{E} = 0$. For convenience and without loss of generality, we omit the tilde on complex permittivity $\tilde{\varepsilon}$ in the following sections. We have

$$\nabla \cdot \left[\varepsilon_\alpha(r)\nabla\Phi\right] = 0, \quad \text{in } \Omega_\alpha, \tag{4}$$

where, the suffixes $\alpha = i, h$ denote the quantities in the inclusion and host regions, respectively. Ω_α $(\alpha = i, h)$ denote the domain of the α-type material. $\varepsilon_i(r) = A + cr^k/i\omega$ is the dielectric graded profile of cylindrical inclusion and ε_h is the complex constant of host material.

In cylindrical coordinates, the governing Eq. (4) of the potential $\Phi_\alpha(r,\varphi)$ becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\varepsilon_\alpha(r) r\frac{\partial\Phi_\alpha}{\partial r}\right] + \frac{1}{r}\frac{\partial}{\partial\varphi}\left[\varepsilon_\alpha(r)\frac{1}{r}\frac{\partial\Phi_\alpha}{\partial\varphi}\right] = 0, \quad \text{in } \Omega_\alpha. \tag{5}$$

In host region, if the external ac electrical field E_a is applied to the composites along the $\hat{x}$-direction, the potential takes the form $\Phi_h(r,\varphi) = -(r + \mathrm{Br}^{-1})E_a\cos\varphi$.

In inclusion region, the local potential can be expressed in terms of a set of basis functions $\{\cos n\varphi\}$, $\Phi_i(r,\varphi) = \sum_{n=0}^{\infty} R_n(r) \times \cos(n\varphi)$. Eq. (5) thus is rewritten as

$$r^2\frac{\partial^2 R_n}{\partial r^2} + r\frac{\partial R_n}{\partial r} + \frac{r^2}{\varepsilon_i(r)}\frac{d\varepsilon_i(r)}{dr}\frac{\partial R_n}{\partial r} - n^2 R_n = 0. \tag{6}$$

In order to solve Eq. (6), we introduce a transformation to Eq. (6), namely, $R_n(r) = r^s g(z)$ and $z = icr^k/A\omega$. We then have

$$(kz)^2\frac{d^2g(z)}{dz^2} + kz\left[2s + k - kz/(1-z)\right]\frac{dg(z)}{dz} + \left[s^2 - n^2 - skz/(1-z)\right]g(z) = 0. \tag{7}$$

Let $s = \pm n$, the Eq. (7) will be the equation of hyper-geometric function

$$z(1-z)\frac{d^2g(z)}{dz^2} + \left[(2s+k)/k - 2(s+k)z/k\right]\frac{dg(z)}{dz} - \frac{s}{k}g(z) = 0. \tag{8}$$

The solution hence can be given by the hyper-geometric function, $F(\alpha_s, \beta_s, \gamma_s; z)$, which is analytic in the whole complex plane except the singular points [15]. The parameters α_s, β_s and γ_s of hyper-geometric function $F(\alpha_s, \beta_s, \gamma_s; z)$ can be given by the following formulas, $\alpha_s + \beta_s + 1 = 2(s+k)/k$, $\alpha_s\beta_s = s/k$, $\gamma_s = (2s+k)/k$. From these formulas, we have $\alpha_s = [2s + k +$

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