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Bandgap properties in locally resonant phononic crystal double panel structures with periodically attached spring–mass resonators

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ABSTRACT

Bandgap properties of the locally resonant phononic crystal double panel structure made of a two-dimensional periodic array of a spring–mass resonator surrounded by n springs (n equals to zero at the beginning of the study) connected between the upper and lower plates are investigated in this paper. The finite element method is applied to calculate the band structure, of which the accuracy is confirmed in comparison with the one calculated by the extended plane wave expansion (PWE) method and the transmission spectrum. Numerical results and further analysis demonstrate that two bands corresponding to the antisymmetric vibration mode open a wide band gap but is cut narrower by a band corresponding to the symmetric mode. One of the regulation rules shows that the lowest frequency on the symmetric mode band is proportional to the spring stiffness. Then, a new design idea of adding springs around the resonator in a unit cell (n is not equal to zero now) is proposed in the need of widening the bandwidth and lowering the starting frequency. Results show that the bandwidth of the band gap increases from 50 Hz to nearly 200 Hz. By introducing the quality factor, the regulation rules with the comprehensive consideration of the whole structure quality limitation, the wide band gap and the low starting frequency are also discussed.

1. Introduction

As is well-known, how to decrease the vibration and reduce the noise is the hot issue that researchers have long been concerning and making efforts to solve. Existing studies show that vibrations mostly propagate along the surrounding structures from the vibration sources of the existing industrial products such as the aircraft, submarine, automobile and so on, by which the noise radiated exactly constitutes the important part of the cabin noise. In addition, the complex sandwich plate structures with the multi-function properties such as light-quality, large-stiffness, excellent shock resistance, good thermal diffusivity and so on are widely applied to the containment structures of the aircraft, submarine and automobile [1–4]. Thus, the researches on the vibration transmission control of the complex sandwich plate structures will play an important role in restraining the structure vibration and reducing the noise in the cabin.

The bringing forward of the phononic crystal concept provides a new idea for the study of the theory of the vibration damping and noise reduction propagated along the structure. Phononic crystal is a kind of periodic composite material with the existence of acoustic/elastic wave band gap which promises a broad application prospect in the field of vibration and noise control. Over the past two decades, the propagation of elastic wave in phononic crystal has attracted a lot of attention which is mainly focused on calculation methods and properties of the band gap such as formation mechanisms and regulation rules, but the application researches particularly on the field of vibration and noise control are still immature. Presently, Brag scattering mechanism [5–8] and locally resonant mechanism [9–12] are developed, and the frequency range of the band gap based on the first mechanism is almost two orders of magnitude higher than that based on the second mechanism [9]. Hence, the studies on the complex sandwich plate structures with the design idea of the traditional locally resonant phononic crystal introduced will provide a new idea for restraining the structure vibration and reducing the noise in the unmanageable low frequency region [13–15] of the cabin of some industrial products.

Presently, bandgap properties of the complex sandwich plate structures with an array of resonant elements periodically mounted have rarely been studied. However, such an idea has been widely implemented in some basic elastic structures such as rods, beams and single plates in recent years. Wang et al. [16] studied the propagation of longitudinal elastic waves in quasi-one-dimensional

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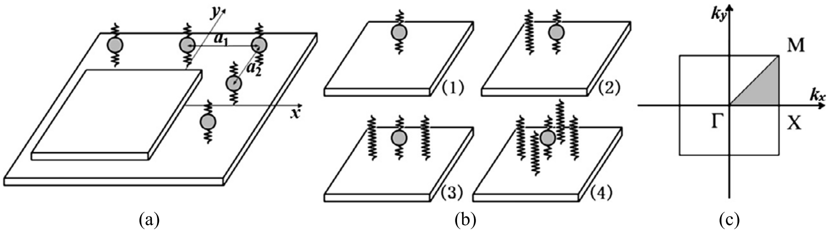

Fig. 1. (a) An infinitely double panel structure with periodically attached spring–mass resonators. (b) The unit cell of the structure (the upper plate is ignored) with n springs surrounded in the case of $n = 0, 1, 2, 4$. (c) The first Brillouin zone.

structure consisting of harmonic oscillators periodically jointed on a slender beam and Yu et al. [17] investigated the flexural vibration in Timoshenko beams with periodically attached local resonators theoretically and experimentally. As for the locally resonant phononic crystal single plate, the so-called filled-in system and stubbed-on system, which are formed by etching holes periodically in a solid matrix plate and then filling them with scatterers as well as stubbing resonant units periodically onto the free surfaces of the plate respectively, were investigated. In Ref. [18] and Ref. [19], the plates with filled-in rubber resonant units and filled-in rubber-coated heavy mass resonant units were researched separately. Similarly to the filled-in structure, the locally resonant phononic crystal single plates constructed by periodically depositing rubber stubs with and without Pb capped on the surface of the plate were studied by Oudich et al. [20]. Besides, Xiao et al. [15] researched the propagation of flexural waves in a locally resonant thin plate made of a two-dimensional periodic array of spring–mass resonators attached on a thin homogeneous plate, which can be regarded as the simplified model of the stubbed-on system. All the papers show that a band gap in the low frequency region can be opened up by the resonant responses of the resonant units and the bands corresponding to the in-plane and the out-of-plane modes are staggered, based on which, a new structure with the three-layered spherical resonant units was proposed and a large sub-wavelength full band gap was opened in Ref. [21]. Based on the researches on the filled-in system and stubbed-on system, Li et al. [22] investigated the propagation characteristics of Lamb waves in a locally resonant phononic crystal single plate whose resonant unit was combined by the filled-in and stubbed-on units. In addition, the formation mechanisms and regulation rules of the band gap in a sandwich plate with a periodic composite core constituted by a square array of elastic cylinders embedded in a solid matrix was researched by Liu et al. [23].

In this paper, we investigate the propagation characteristics of flexural waves in a locally resonant double panel system consisting of a two-layer uniform thin plate with periodically attached spring–mass resonators in the cavity, each of which is surrounded by n springs. At first, for the proposed locally resonant phononic crystal double panel structure without spring surrounded (i.e. $n = 0$), the accurate band structure is studied. Then, the formation mechanisms and regulation rules of the band gap of the $n = 0$ system are researched in detail, based on which the improved system with n springs surrounded is proposed in the need of widening the band gap and lowering the starting frequency. The new bandgap properties are discussed and presented. Besides, the relations between the minimum value of the total stiffness and the number, the location of the additional springs are researched. All the results are expected to be of theoretic significances and engineering application prospects in the field of vibration and noise reduction.

2. Model and method

As the model shown in Fig. 1(a), an infinitely double panel structure with periodically attached resonators is taken into consideration. What should be noted in this paper is that the effect of the air between the two plates is ignored. The parameters of the isotropic plates are separately defined as follows: the mass density, Young's modulus, Poisson's ratio and thickness of the lower plate are ρ_1, E_1, μ_1 and h_1 while the ones of the upper plate are ρ_2, E_2, μ_2 and h_2. In addition, each resonator of the structure consists of two springs with the same stiffness k_R and a mass m_R surrounded by n springs with the uniform stiffness k_S (Fig. 1(b) shows the unit cell of the structure (the upper plate is ignored) in the case of $n = 0, 1, 2, 4$). In this paper, the rectangular lattice, whose basis vectors are $\boldsymbol{a_1} = (a_1, 0)$ and $\boldsymbol{a_2} = (0, a_2)$, is adopted for numerical calculation. So, as shown in Fig. 1(a), we define the neutral surface of the lower thin plate as the XY plane which is used as the basis of building the coordinate system, and array each resonator between two plates at the point denoted by the basis vectors $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$:

$$\boldsymbol{R} = \overline{m}\boldsymbol{a_1} + \overline{n}\boldsymbol{a_2}, \tag{1}$$

where $\overline{m}$ and $\overline{n}$ are integers.

For the structure proposed in this paper, the resulting finite element model with the damping term not considered can be represented as follows:

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{w} = \mathbf{F}, \tag{2}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the structural stiffness and mass matrices, ω represents the circular frequency of structural vibration, $\mathbf{w}$ and $\mathbf{F}$ denote the displacement and external load vectors.

It is well known that the periodic boundary conditions according to the Bloch–Floquet theorem [20] should be used for the interfaces between the nearest unit cells:

$$\mathbf{w}(x + a_1, y + a_2) = \exp\left(i(k_x a_1 + k_y a_2)\right)\mathbf{w}(x, y), \tag{3}$$

where (x, y) denotes the position vector, k_x and k_y are the components of the Bloch wave vector limited in the irreducible first Brillouin zone (1BZ), as shown in Fig. 1(c).

In the present work, based on all the formulas presented above, the band structure of the infinite system and the vibration transmittance of the corresponding finite system with countable periodic array of the unit cell are calculated, respectively. Here, the four node quadrilateral plate element is adopted as the discussed finite element. Besides, the spring element and mass element are used to describe the resonator. Besides, the size of the plate meshing must be adapted to the variation behavior of the solution. As is well-known, the smaller the mesh size is used, the better the convergence of the computation can be obtained, but the longer the calculation time has to be used. So it is important to choose an adaptive mesh that allows us to have sufficiently good convergence in an acceptable calculation time.

Fig. 2 shows the finite element mesh division of a unit cell. For the divided elements, according to equation (3), the nodes on the right and up boundaries of the unit cell can be replaced by the ones on the left and down boundaries. As a consequence, the node number and the matrix dimension are reduced in equation (2).