



Quantum remnants in the classical limit



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ABSTRACT

We analyze here the common features of two dynamical regimes: a quantum and a classical one. We deal with a well known semi-classic system in its route towards the classical limit, together with its purely classic counterpart. We wish to ascertain i) whether some quantum remnants can be found in the classical limit and ii) the details of the quantum-classic transition. The so-called mutual information is the appropriate quantifier for this task. Additionally, we study the Bandt–Pompe's symbolic patterns that characterize dynamical time series (representative of the semi-classical system under scrutiny) in their evolution towards the classical limit.

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1. Introduction

Information measures (IM) (see as examples [1–5], and references therein) are of utility in the analysis of a given time series' (TS) underlying dynamics. The standard Boltzmann–Gibbs–Shannon entropy is the best known IM

$$S = - \sum_{i=1}^n p_i \ln p_i, \quad (1)$$

but of course, there exist other entropy-based quantifiers that may be profitably employed according to the researcher's goals. We will apply one of these quantifiers, the Mutual Information [6], to a celebrated semiclassical system in its route towards the classical limit [7,8]. It is firmly established that the system's dynamics exhibits a quantum zone, a transition one, and a classic region [8]. This dynamics exhibits regular features plus i) some chaotic ones and ii) other type of dynamics as well that, although not chaotic, exhibit complex features [8]. This system has attracted intensive attention both from the dynamic [8] and the statistical points of view [9–11]. In this effort we wish to find out *what are the common features observed during the transition process both in the quantum regime and in the other ones*. In order to do so we have first of all

to consider the issue of how to draw relevant information from a time series (TS) [12]. Data are often contaminated by stochastic noise [13,14]. Accordingly, distinct extraction-methodologies produce different quality's degrees. In this work we apply Bandt and Pompe's method for the purpose [15]. Note that an early symbolic approach is that of Beck and Graudenz [16]. With it we get a convenient probability distribution function (PDF) linked to the time series of interest.

This methodology converts the time series into a sequence of characteristic “patterns” (symbols) that can be easily visualized, characterized, and analyzed. One can then identify the patterns (symbols) of the quantum regime and investigate whether some of them “survive” in the classical limit.

Our semiquantum system and its classical analog are described in Section 2. In Section 3.1, Bandt and Pompe's symbolic approach is briefly revisited. Also, basic features of the Mutual Information concept are remembered in Section 3.2. Our present results are presented in Section 4. Finally, some conclusions are drawn in Section 5.

2. The classical-quantum transition

Such transition constitutes an important physics topic. Quantum mechanics' classic limit (CLQM) is a frontier issue [17–21] and originating much work, as, for instance, [17,18] and references therein. In this vein, people regard “quantum” chaotic motion as deserving concentrated interest. Recent progress can be consulted in [22]. Also, generalized uncertainty principles (GUP)

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[23,24] attract attention. Zurek and others Habib [19–21] have discussed how our classical reality can emerge from the quantum world.

The present authors will work here from a semiclassical viewpoint. We may speak, for starters, of both the WKB and Born–Oppenheimer approximations. Our interest lies in the bipartite model of Bonilla and Guinea [25] and Cooper et al. [7], to which Kowalski et al. [1,26,8] have made extensive contributions. In the bipartition one system is of quantal nature while the remaining one is classical. The quantum part contributes in small measure to the overall picture [8]. Such scenario is illustrated by a two-level system's interaction with an EM-field within a cavity, Bloch equations, or nuclear collective modes. The composite model of Cooper et al. represents the production of charged meson pairs [7,8].

2.1. The Hamiltonian

The pertinent Hamiltonian mixes quantum variables with classical ones. It reads

$$\hat{H} = \frac{1}{2} \left(\frac{\hat{p}^2}{m_q} + \frac{P_A^2}{m_{cl}} + m_q \omega^2 \hat{x}^2 \right), \quad (2)$$

and we emphasize the presence of the product $\omega^2 \hat{x}^2$ in the third term. In Eq. (2) we have two quantum operators \hat{x} and \hat{p} . Moreover, A and P_A are classical quantities (canonically conjugate). As just stated, we encounter in the third term a nonlinear interaction involving $\omega^2 = \omega_q^2 + e^2 A^2$, where ω_q is a frequency. The masses m_q and m_{cl} correspond, respectively, to the quantum and classical components of our bipartite system. Kowalski et al. have shown [26] that dealing with this Hamiltonian leads to an (autonomous) nonlinear coupled differential equations' system.

2.2. Nonlinear system of coupled differential equations

We face [26]

$$\begin{aligned} \frac{d\langle \hat{x}^2 \rangle}{dt} &= \frac{\langle \hat{L} \rangle}{m_q}; & \frac{d\langle \hat{p}^2 \rangle}{dt} &= -m_q \omega^2 \langle \hat{L} \rangle \\ \frac{d\langle \hat{L} \rangle}{dt} &= 2 \left(\frac{\langle \hat{p}^2 \rangle}{m_q} - m_q \omega^2 \langle \hat{x}^2 \rangle \right) \\ \frac{dA}{dt} &= \frac{P_A}{m_{cl}} & \frac{dP_A}{dt} &= -e^2 m_q A \langle \hat{x}^2 \rangle \\ \hat{L} &= \hat{x}\hat{p} + \hat{p}\hat{x}. \end{aligned} \quad (3)$$

The above system of equations can be derived from i) Ehrenfest's relations for quantum variables and ii) canonical Hamilton's equations for classical quantities [26]. The analysis of the classic limit is helped by consideration of the classical partner of (2) (where all variables are of classic mature). Hamilton's equations lead one to a purely classic partner of the system (3). See, for more details, [26].

2.3. Dealing with our nonlinear ODE system

One conveniently reach the classic limit as the mathematical limit of a quantity E_r that could be thought of as a “relative energy” [8]

$$E_r = \frac{|E|}{I^{1/2} \omega_q} \rightarrow \infty, \quad (4)$$

with E the total system's energy. The quantity I is a motion-invariant of (3), related to Heisenberg's celebrated Principle

$$I = \langle \hat{x}^2 \rangle \langle \hat{p}^2 \rangle - \frac{\langle \hat{L} \rangle^2}{4} \geq \frac{\hbar^2}{4}. \quad (5)$$

We deal with the system (3) via numerical methodologies, for multiple sets of initial conditions. Things are best described by plotting relevant quantities against the associated multiple values of E_r in the range $[1, \infty]$.

It is firmly established in the Literature (see, for instance [26]) that i) if $E_r = 1$, the quantum component of the bipartite system absorbs the whole of the energy $E = I^{1/2} \omega_q$ while, ii) quantal and classical variables are localized at the fixed point ($\langle \hat{x}^2 \rangle = I^{1/2} / m_q \omega_q$, $\langle \hat{p}^2 \rangle = I^{1/2} m_q \omega_q$, $\langle \hat{L} \rangle = 0$, $A = 0$, $P_A = 0$) [26], and, because $A = 0$, no coupling between the two systems ensues. For $E_r \sim 1$ our composite system becomes an “almost” quantum one of quasi-periodic dynamics [8].

The time series that we will discuss here is indeed an E_r -series. When E_r grows, the quantum features tend to vanish in a rather rapid fashion and one speaks of entering a semiclassical zone. A particular value E_r^{cl} signals that the solutions to Eqs. (3) start resembling classicality [8]. Convergence of the solutions for Eqs. (3) to the classical ones is reached and, for large E_r -values, our composite systems is a classic one. One considers the time series' sector $1 < E_r < E_r^{cl}$ as a semi-classical region. In such a zone we distinguish a special value $E_r = E_r^P$. There, one can speak of emergence of chaos [26].

3. Our present approach

As stated above, we will associate our physical problem to a pseudo time-series (PTS) in which **what unfolds is not time** but the value of E_r , in the E_r -progress towards the classical limit (including the classical analog of the Eqs. (3)). That is, we consider multiple realizations of our system, each of them with different initial conditions chosen in such a way that different E_r -values ensue. There is a different E_r -value for each realization. This “ E_r -evolution” is, of course, not a unitary one. Our time series are E_r -series, NOT temporal ones. We do not study time evolution but E_r -one.

To obtain the PTS-associated probability distribution functions we use the rather popular Bandt and Pompe's (BP) approach [15]. With these PDFs we compute the Mutual Information (MI) for each E_r value. These MIs constitute our main research tool here, together with the TS-patterns (symbols) that the Bandt–Pompe helpfully yields.

3.1. Probability distributions based on the Bandt–Pompe's methodology

We employ here the BP procedure [15] in order to obtain the probability distribution P associated with our E_r -(pseudo) time series. The full details are given, for instance, in [1,15,27] and will not be repeated here. One assigns to each E_r -value a D -dimensional vector of preceding times. This vector is converted into a unique symbol or pattern.

3.2. Forbidden patterns

An interesting feature of the BP treatment is the existence of so-called *forbidden patterns* (see [28,29] and references therein).

For deterministic one dimensional maps, it has been conclusively shown that not all the possible ordinal patterns (as defined using Bandt and Pompe's methodology) can be effectively materialized into orbits, which in a sense makes these patterns forbidden (see [28,29] and references therein). We insist: this is an established fact, not a conjecture. The existence of these forbidden ordinal patterns becomes a persistent feature, a “new” dynamical property. For a fixed pattern-length (embedding dimension D) the number of forbidden patterns of a time series (unobserved patterns) is independent of the series' length N . It must be realized that this independence does not characterize other properties of

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