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Green's functions of one-dimensional quasicrystal bi-material with piezoelectric effect

Liangliang Zhang [b,c], Di Wu [b], Wenshuai Xu [a], Lianzhi Yang [d], Andreas Ricoeur [e], Zhibin Wang [e], Yang Gao [a,*]

[a] College of Science, China Agricultural University, Beijing 100083, PR China
[b] College of Engineering, China Agricultural University, Beijing 100083, PR China
[c] Sinomatech Wind Power Blade Co., Ltd, Beijing 100092, PR China
[d] Civil and Environmental Engineering School, University of Science and Technology Beijing, Beijing 100083, PR China
[e] Institute of Mechanics, University of Kassel, 34125 Kassel, Germany

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ABSTRACT

Based on the Stroh formalism of one-dimensional quasicrystals with piezoelectric effect, the problems of an infinite plane composed of two different quasicrystal half-planes are taken into account. The solutions of the internal and interfacial Green's functions of quasicrystal bi-material are obtained. Moreover, numerical examples are analyzed for a quasicrystal bi-material subjected to line forces or line dislocations, showing the contour maps of the coupled fields. The impacts of changing material constants on the coupled field components are investigated.

1. Introduction

Quasicrystals (QCs) were found initially by Shechtman et al. [1]. Being a new solid structure, they differ from ordinary crystals and non-crystals. QCs are promising materials with light mass and high strength, possessing great development trends, being adopted progressively by industry, for instance, in automobile, aerospace or power source technologies. On account of the quasi-periodic symmetry of QCs, concepts of the high-dimensional space have been introduced instead of the classical crystallographic theory. The phonon field represents the lattice vibrations in QCs, and the phason field depicts its quasi-periodic rearrangement of atoms; both these fields can be used to describe the elasticity of QCs. One-dimensional (1D) QCs exhibit just one quasi-periodic axis, while the perpendicular plane reveals classical crystalline properties. A large number of papers have come off the press on the elasticity of QCs. Chen et al. [2] analyzed three-dimensional (3D) elastic problems of 1D hexagonal QCs by quasi-harmonic functions. Gao [3] derived the exact solutions for deep beams of 1D QCs without any presupposes. Li and Li [4] obtained the 3D thermoelastic general solutions of 1D QCs. Li and Fan [5] developed the stress potential function theory for plane problems of icosahedral QCs. Moreover, some scholars have studied defect problems [6–8] of QCs.

The researches on Green's functions of anisotropic elastic solids [9,10] have been carried out early, then Green's functions of multi-field coupled problems have been analyzed. Pan [11] analyzed 3D Green's functions of magneto-electro-elastic materials under point loadings by the extended Stroh formalism. Sevostianov et al. [12] derived Green's functions of piezoelectric crystals with 622 hexagonal symmetry subjected to point forces and point electric charges. Moreover, Green's functions of nonlinear [13] and dynamic systems [14] have been derived. In accordance with the Stroh formalism and conforming mapping, Qin [15] derived the solutions of Green's functions of defective electro-magneto-thermo-elastic solids exposed to thermal loading. Most of the engineering structures contain internal interfaces. When these structures with some defects, such as dislocations, cracks or holes, are exposed to multi-physical loads, failure and fracture may occur. So it is of crucial importance in structural design to study these defects. Green's functions have been extended to thermoelastic [16], thermo-electro-elastic [17], electro-magneto-thermo-elastic [18] and other bi-materials [19,20]. Furthermore, the Green's function method has also been identified as an important approach in the studies of elastic theory of QCs [16,21,22].

* Corresponding author.
E-mail address: gaoyangg@gmail.com (Y. Gao).

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Because of the coupling effects between electric and phonon, phason and electric fields, analytical solutions of QCs with piezoelectric effect are hard to be obtained. The piezoelectric effect of QCs [23–31] has been studied to a certain degree, however by far not sufficiently. Due to its sententiousness, QCs with piezoelectric effect will be investigated in this paper by fusing the theories of QCs and piezoelectric materials together and using a generalized Stroh formalism.

Various boundary value problems, specifically fracture problems, can be analyzed in depth on basis of the solutions of Green's functions. The analytical solutions are favorable for engineering applications, fostering a deep understanding of the mechanical behavior of the materials. In the present paper, the internal and interfacial Green's functions of a 1D QC bi-material with piezoelectric effect are presented. By using the Stroh formalism, the coupled fields of 1D QC bi-materials are obtained for different cases. Numerical results are given and the mechanical behavior under the influence of different coupling constants is analyzed.

2. Stroh formalism of QCs with piezoelectric effect

In the linear elastic theory of QCs, the geometry equations, constitutive relations and equilibrium equations of 1D piezoelectric QCs without body force can be expressed as

$$\varepsilon_{mj} = (u_{m,j} + u_{j,m})/2, \quad w_{3j} = w_{3,j}, \quad E_j = -\phi_{,j}, \tag{1}$$

$$\begin{aligned} &\sigma_{mj} = C_{mjvl}\varepsilon_{vl} + R_{mj3l}w_{3l} - e^{(1)}_{lmj}E_l, \\ &H_{3j} = R_{vl3j}\varepsilon_{vl} + K_{3j3l}w_{3l} - e^{(2)}_{l3j}E_l, \\ &D_j = e^{(1)}_{jvl}\varepsilon_{vl} + e^{(2)}_{j3l}w_{3l} + \xi_{jl}E_l, \end{aligned} \tag{2}$$

$$\sigma_{mj,j} = 0, \quad H_{3j,j} = 0, \quad D_{j,j} = 0, \tag{3}$$

where m, j, v, $l = 1, 2, 3$, and the denotation "," represents the derivative operation for the space variables. u_m, w_3 and ϕ are the phonon displacements, phason displacement, and electric potential, respectively, and the atom arrangement is periodic in the x_1–x_2 plane and quasi-periodic in the x_3-axis; σ_{mj} and ε_{mj} are the phonon stresses and strains, respectively; H_{3j} and w_{3j} are the phason stresses and strains, respectively; D_j and E_j are the electric displacements and electric fields, respectively, and the polarization direction is along the x_3-axis; C_{mjvl} and K_{3j3l} are the elastic constants in the phonon and phason fields, respectively; R_{mj3l} represent the phonon–phason coupling elastic constants; $e^{(1)}_{jvl}$ and $e^{(2)}_{j3l}$ are the piezoelectric constants in the phonon and phason fields, respectively; ξ_{mj} are the permittivity constants.

For plane problems, only the x_1–x_3 plane is considered, so all the field components are independent of x_2. The constitutive relations and equilibrium equations of piezoelectric QCs are rewritten in a compressed notation as

$$\sigma_{\alpha j} = E_{\alpha jKl}u_{K,l}, \tag{4}$$

$$\sigma_{\alpha 1,1} + \sigma_{\alpha 3,3} = 0, \tag{5}$$

where α, $K = 1, 2, 3, 4, 5$,

$$\begin{aligned} &\mathbf{u} = [u_1, u_2, u_3, w_3, \phi]^{\mathrm{T}}, \quad \sigma_{\alpha j} = \sigma_{\alpha j} \quad (\alpha = 1, 2, 3), \\ &\sigma_{\alpha j} = H_{3j} \quad (\alpha = 4), \qquad \sigma_{\alpha j} = D_j \quad (\alpha = 5), \\ &E_{mjvl} = C_{mjvl}, \quad E_{mj4l} = R_{mj3l}, \quad E_{mj5l} = e^{(1)}_{lmj}, \quad E_{4jvl} = R_{vl3j}, \\ &E_{5jvl} = e^{(1)}_{jvl}, \quad E_{4j4l} = K_{3j3l}, \quad E_{4j5l} = e^{(2)}_{l3j}, \quad E_{5j4l} = e^{(2)}_{j3l}, \\ &E_{5j5l} = -\xi_{jl}, \end{aligned} \tag{6}$$

where the superscript "T" stands for the matrix transpose. Yang et al. [32] introduced the generalized stress vector $\boldsymbol{\varphi}$ representing the stress components $\sigma_{\alpha 1}$ and $\sigma_{\alpha 3}$,

$$\sigma_{\alpha 1} = -\varphi_{\alpha,3}, \quad \sigma_{\alpha 3} = \varphi_{\alpha,1}, \tag{7}$$

and then derived the general solution of 1D piezoelectric QCs for plane problems

$$\mathbf{u} = \mathbf{A}\mathbf{f}(z_\alpha) + \overline{\mathbf{A}}\,\overline{\mathbf{f}(z_\alpha)}, \ \boldsymbol{\varphi} = \mathbf{B}\mathbf{f}(z_\alpha) + \overline{\mathbf{B}}\,\overline{\mathbf{f}(z_\alpha)}, \tag{8}$$

where the bar denotes the conjugate complex quantity, and

$$\begin{aligned} &\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5], \quad \mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4, \mathbf{b}_5], \\ &\mathbf{f}(z_\alpha) = \left[f_1(z_1), f_2(z_2), f_3(z_3), f_4(z_4), f_5(z_5)\right]^{\mathrm{T}}, \\ &z_\alpha = x_1 + p_\alpha x_3. \end{aligned} \tag{9}$$

It is noted that the Stroh's sextic formalism for anisotropic elastic materials [33] is extended to a tenth-order formalism for 1D piezoelectric QCs. The mathematical formulations of the Stroh formalism for 1D piezoelectric QCs keep the same form as those for anisotropic elastic materials, but with different orders. **A** and **B** are two constant matrices. $\mathbf{a}_\alpha$ and $\mathbf{b}_\alpha$ are the homologous eigenvectors. $f_\alpha(z_\alpha)$ are the arbitrary functions depending on z_α. p_α are the distinct eigenvalues of the following equation, i.e. multiple eigenvalues are excluded,

$$\mathbf{N}\boldsymbol{\xi} = p\boldsymbol{\xi}, \ \boldsymbol{\xi} = [\mathbf{a}, \mathbf{b}]^{\mathrm{T}}, \tag{10}$$

where **N** is a 10×10 matrix,

$$\begin{aligned} &\mathbf{N} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{N}_3 & \mathbf{N}_1^{\mathrm{T}} \end{bmatrix}, \\ &\mathbf{N}_1 = -\mathbf{T}^{-1}\mathbf{R}^{\mathrm{T}}, \quad \mathbf{N}_2 = \mathbf{T}^{-1}, \quad \mathbf{N}_3 = \mathbf{R}\mathbf{T}^{-1}\mathbf{R}^{\mathrm{T}} - \mathbf{Q}, \end{aligned} \tag{11}$$

the matrices **Q**, **R** and **T** are 5×5 real matrices as follows

$$Q_{\alpha K} = E_{\alpha 1K1}, \quad R_{\alpha K} = E_{\alpha 1K3}, \quad T_{\alpha K} = E_{\alpha 3K3}, \tag{12}$$

and

$$\begin{aligned} &p_{\alpha+5} = \bar{p}_\alpha, \quad \operatorname{Im} p_\alpha > 0, \quad \mathbf{a}_{\alpha+5} = \bar{\mathbf{a}}_\alpha, \\ &\mathbf{b}_{\alpha+5} = \bar{\mathbf{b}}_\alpha \quad (\alpha = 1, 2, 3, 4, 5), \end{aligned} \tag{13}$$

where Im represents the imaginary part. For obtaining the solutions in a real form, the following matrices are introduced

$$\mathbf{S} = \mathrm{i}\big(2\mathbf{A}\mathbf{B}^{\mathrm{T}} - \mathbf{I}\big), \quad \mathbf{H} = 2\mathrm{i}\mathbf{A}\mathbf{A}^{\mathrm{T}}, \quad \mathbf{L} = -2\mathrm{i}\mathbf{B}\mathbf{B}^{\mathrm{T}}, \tag{14}$$

where $\mathrm{i} = \sqrt{-1}$. Thus the problem of elasticity of 1D QCs with piezoelectric effect can be transformed into that of calculating the characteristic matrices, characteristic values and the complex vector $\mathbf{f}(z_\alpha)$ with the given boundary conditions.

3. Green's functions of 1D QC bi-materials with piezoelectric effect

In this section, Green's functions will be extended to QC bi-materials with piezoelectric effect. The internal and interfacial Green's function solutions are obtained.

3.1. Interior Green's function solutions

The problem of an infinite plane which is composed of two different piezoelectric QCs is taken into account. The line forces $\hat{\mathbf{p}}$ and line dislocations $\hat{\mathbf{b}}$ are acting at the interior point $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_3)$ as shown in Fig. 1. The boundary conditions can be expressed as

$$\mathbf{u}^+ = \mathbf{u}^-, \quad \boldsymbol{\varphi}^+ = \boldsymbol{\varphi}^-, \quad \text{along the interface } x_3 = 0, \tag{15}$$

$$\oint_C \mathrm{d}\boldsymbol{\varphi}^- = \hat{\mathbf{p}}, \quad \oint_C \mathrm{d}\mathbf{u}^- = \hat{\mathbf{b}} \quad \text{for any closed curve } C \text{ enclosing the point } \hat{\mathbf{x}}, \tag{16}$$

$$\sigma_{\alpha j} \to 0, \quad \text{at infinity}, \tag{17}$$