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Excitonic entanglement of protected states in quantum dot molecules

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The entanglement of an optically generated electron–hole pair in artificial quantum dot molecules is calculated considering the effects of decoherence by interaction with environment. Since the system evolves into mixed states and due to the complexity of energy level structure, we use the negativity as entanglement quantifier, which is well defined in $D \otimes D'$ composite vector spaces. By a numerical analysis of the non-unitary dynamics of the exciton states, we establish the feasibility of producing protected entangled superposition by an appropriate tuning of bias electric field, F. A stationary state with a high value of negativity (high degree of entanglement) is obtained by fine tuning of F close to a resonant condition between indirect excitons. We also found that when the optical excitation is approximately equal to the electron tunneling coupling, $\Omega/T_e \sim 1$, the entanglement reaches a maximum value. In front of the experimental feasibility of the specific condition mentioned before, our proposal becomes an useful strategy to find robust entangled states in condensed matter systems.

1. Introduction

Semiconductor quantum dot molecules (QDMs) driven by coherent pulses have been extensively suggested as promising candidates for physical implementation of solid state in quantum information (QI) processing [1,2]. The possibility of selective control of the electronic occupation and a controllable energy spectrum are the major characteristics which allow QDMs to be viable for the realization of universal quantum computation [3]. The flexibility of quantum dots (QDs) as quantum information systems has been proven by successful implementation of controllable operations on charge [4] and optical qubits [5,6].

In addition to a feasible qubit physical implementation, quantum entanglement is a fundamental nonlocal resource for quantum computation and communication. Although there are several theoretical approaches proposing methods for direct measurement of entanglement [7,8], its experimental quantification remains a challenge. Semiconductor quantum dots optically addressed have proven to be a very appropriate system to investigate quantum correlations [9], providing required conditions to investigate and demonstrate entanglement between spin–photon [10], photons emitted from recombination of optical excitations [11] and electron–hole pairs [12]. Optically induced entanglement of excitons has been performed in single QDs [13] and QDMs [14], where interaction between particles and interdot tunneling are the key mechanisms for the formation of entangled states. Several theoretical calculations have proposed different strategies to obtain exciton states in QDMs with a high degree of entanglement [15,16,12]. Also, the electron–hole entanglement can be efficiently tuned and optimized through the proper choice of interdot separation, QD asymmetry and by the action of an electric field applied in the growth direction [17,18].

In ideal conditions, the QDM undergoes unitary evolution, the quantum entanglement is not affected by decoherence and its calculation can be easily performed by using Von Neumann entropy [12,17]. However, a system is unavoidable coupled with the environment which leads to degradation of quantum coherence. In these conditions, the system evolution is non-unitary and the decoherence effects cause deterioration of the entanglement and will be detrimental for production, manipulation and detection of entangled states. For QDMs the main decoherence channels are the radiative decay of excitons and exciton pure dephasing which is important even at low temperatures [19]. The definition of a computable quantifier of entanglement for general mixed states in open quantum systems has been a challenge for the last decades [20]. Among the diverse bipartite entanglement measurements, entanglement of formation [21] and concurrence [22] are well-defined and extensively used to evaluate the entanglement of mixed bipartite in $2 \otimes 2$ systems. An alternative measure for mixed states is negativity, $\mathcal{N}(\rho)$, first proposed by Vidal and Werner [23],

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which overcomes the limitations of other measurements and allows to calculate the degree of entanglement for general $D \otimes D'$ composite vector spaces [24]. It is important to mention that, in the context of QDMs, electron–hole entanglement was previously investigated using the von Neumann entropy ignoring the essential effects of decoherence [17,18,12].

In this paper, we investigate the entanglement degree of electron–hole pairs created in QDMs by the incidence of coherent radiation considering spontaneous exciton decay and pure dephasing as main decoherence sources. The degree of entanglement in the asymptotic regime is evaluated through the negativity $\mathcal{N}(\rho)$ as a function of controllable physical parameters, exploring the conditions which maximize the entanglement degree. Our results show that the system evolves to asymptotic states induced by dissipative mechanisms which are superposition of indirect exciton states. For experimentally accessible conditions, such states have long lifetimes [25] and high degree of entanglement.

2. Description of the system

We consider a QDM composed by two asymmetric QDs vertically aligned and separated by a barrier of width d. The electron–hole occupation is controlled by the interplay of tunneling coupling, optical excitation and gate potentials. Due to structural asymmetry of QDs, the energy levels of each carrier become resonant for specific values of an external electric field F applied along the growth direction of the QDM. A careful growth engineering along with the natural QDM asymmetry allows the design of samples with selective tunneling of electrons or holes [26].

In order to investigate the entanglement between electron and hole, we model our system using a composite particle position basis $|X^{e_T,h_T}_{e_B,h_B}\rangle = |^{e_T}_{e_B}\rangle \otimes |^{h_T}_{h_B}\rangle$, where $e_{T(B)}$, $h_{T,(B)}$ represent the occupation number of electrons and holes in each level in the top (T) or bottom (B) QD, respectively [27,14,28]. The QDM is driven by a low-intensity continuous wave laser, such that only the ground-state exciton can be formed. The occupation of the electron or the hole in each QD should be 0 or 1, where the value 0 (or 1) represents the absence (presence) of the carrier in the QD. For this reason, in our model the computational basis for each subsystem (electron or hole) has dimension 4. For instance, the ket $|X^{10}_{01}\rangle$ represents an indirect exciton state, with one electron occupying the top QD and one hole in the bottom QD. Thus, the QD position index encodes the information of a specific quantum state and the complete basis set of the composite system $\mathcal{H} = \mathcal{H}_e \otimes \mathcal{H}_h$ is composed by 16-states. Considering only the optical active transitions, tunneling of electrons and holes, and assuming that the QDM is initially uncharged, the QDM system is described using the composite basis $\{|X^{00}_{00}\rangle, |X^{00}_{11}\rangle, |X^{10}_{01}\rangle, |X^{11}_{00}\rangle, |X^{01}_{10}\rangle, |X^{11}_{11}\rangle\}$. Under electric-dipole and rotating-wave approximations and after removing the time dependence, the resulting Hamiltonian is given by

$$H = \begin{pmatrix} 0 & \Omega & 0 & \Omega & 0 & 0 \\ \Omega & \delta^{00}_{11} & T_e & V_f & T_h & \Omega \\ 0 & T_e & \delta^{10}_{01} - \Delta_F & T_h & 0 & 0 \\ \Omega & V_f & T_h & \delta^{11}_{00} & T_e & \Omega \\ 0 & T_h & 0 & T_e & \delta^{01}_{10} + \Delta_F & 0 \\ 0 & \Omega & 0 & \Omega & 0 & \delta^{11}_{11} + V_{XX} \end{pmatrix}, \quad (1)$$

where $\delta^{e_T,h_T}_{e_B,h_B}$ is the detuning of the incident laser and the exciton states, $\Delta_F = eFd$ is the Stark energy shift on the indirect excitons [29], being d the barrier thickness between the QDs. $T_{e(h)}$ describes the single-particle interdot tunneling for electrons (holes), V_F is the interdot coupling between direct excitons via the Föster mechanism, and V_{XX} accounts for direct Coulomb binding energy between two excitons, one located on each dot [30,31]. The optical coupling is given by the parameter Ω, which depends on the laser intensity and oscillator strength of allowed optical transitions.

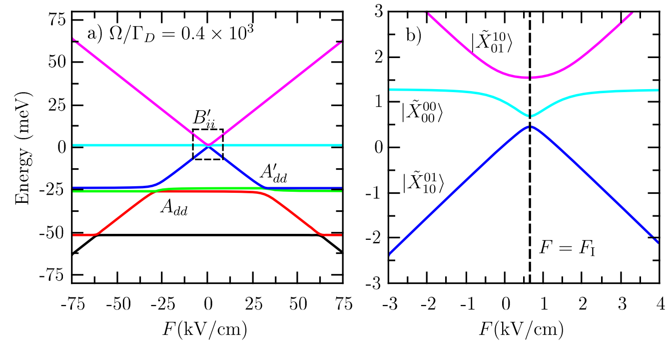

Fig. 1. (Color online.) Left panel: Energy eigenvalues of Hamiltonian (1) as a function of electric field F, for $\Omega/\Gamma_D = 0.4 \times 10^3$, $V_f/\Gamma_D = 8$, $T_e/\Gamma_D = 20T_h/\Gamma_D = 0.2 \times 10^3$, $V_{XX}/\Gamma_D = -0.5 \times 10^3$ and $d = 8.4$ meV. Right panel: Zoom of the dashed area in panel a), showing the energy eigenvalues near to $F = F_I$ which corresponds to the anticrossing condition between the indirect exciton states. For simplicity, we label the exciton vacuum $|\tilde{X}^{00}_{00}\rangle$ and indirect exciton states ($|\tilde{X}^{01}_{10}\rangle$, $|\tilde{X}^{10}_{01}\rangle$) levels according to their dominant character for large F.

For numerical calculations we use the exciton bare energies and coupling parameters for an InAs/GaAs QDM given in Refs. [27,26]. All the energy parameters are scaled in units of effective decoherence rate Γ_D, which will be defined below. Fig. 1a) shows the eigenvalues of Hamiltonian (1) as a function of electric field F, for $\Omega/\Gamma_D = 0.4 \times 10^3$ and $d = 8.4$ meV. The energy levels consist of direct exciton states, weakly dependent on the electric field F, and indirect exciton states, which are strongly dependent with F. Direct and indirect exciton states are coupled by tunneling and exhibit large anticrossings. At well-defined values of F, it is important to note the arising of small anticrossings, labeled as A and B in Fig. 1a). These anticrossings correspond to coupling between exciton states of the same character. For instance, the anticrossings identified as A_{dd} and A'_{dd} are related to the coupling between direct exciton states (intradot excitons). From here on, we focus our attention at anticrossing labeled as B_{ii}. In Fig. 1b), we show the detailed structure of anticrossing B_{ii} which involves the two indirect exciton states and the vacuum state. The indirect exciton states (interdot excitons) $|\tilde{X}^{01}_{10}\rangle$ and $|\tilde{X}^{10}_{01}\rangle$ are effectively coupled at field value $F = F_I$. This particular value of the electric field is obtained through of the resonance condition between indirect bare excitons: $\delta^{10}_{01} - \Delta_F = \delta^{01}_{10} + \Delta_F$.

In the context of QI processing in QD systems, indirect excitons have been proposed as long-lived and optically controllable qubits, as pointed out in Ref. [27]. In our system, the two indirect states are not directly coupled to each other, as can be verified from Hamiltonian (1). However, as was shown in detail in Ref. [27] these states are in fact effectively coupled, which is in turn the result of the combined action of both, electron and hole tunnelings. This can be checked by projecting out the direct exciton states of the total Hamiltonian to obtain the effective coupling between indirect excitons which is found to be proportional to T_eT_h. Using the same procedure it is found that the effective coupling between the vacuum state and indirect excitons is nearly proportional to $\Omega T_{e(h)}$.

3. Entanglement in QDM

3.1. Closed QDM system

The anticrossings of type A and B are directly related to the emergence of strong electron–hole entanglement as shown in Refs. [12,18]. In closed quantum systems, this assertion is