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Noise-induced absorbing phase transition in a model of opinion formation

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ABSTRACT

In this work we study a 3-state (+1, −1, 0) opinion model in the presence of noise and disorder. We consider pairwise competitive interactions, with a fraction p of those interactions being negative (disorder). Moreover, there is a noise q that represents the probability of an individual spontaneously change his opinion to the neutral state. Our aim is to study how the increase/decrease of the fraction of neutral agents affects the critical behavior of the system and the evolution of opinions. We derive analytical expressions for the order parameter of the model, as well as for the stationary fraction of each opinion, and we show that there are distinct phase transitions. One is the usual ferro–paramagnetic transition, that is in the Ising universality class. In addition, there are para-absorbing and ferro-absorbing transitions, presenting the directed percolation universality class. Our results are complemented by numerical simulations.

1. Introduction

The study of dynamics of opinion formation is nowadays a hot topic in the Statistical Physics of Complex Systems, with a considerable amount of papers published in the last years (see [1–4] and references therein). Even simple models can exhibit an interesting collective behavior that emerges from the microscopic interaction among individuals or agents in a given social network. Usually those models exhibit nonequilibrium phase transitions and rich critical phenomena, which justifies the interest of physicists in the study of opinion dynamics [1–11].

In the last few years, a recent attention has been done to the kinetic exchange opinion models (KEOM) [7–9,12], inspired in models of wealth exchange [13–15]. The LCCC model was the first one to consider kinetic exchanges among pairs of agents that present continuous states (opinions) [7]. In this case, the model presents a continuous symmetry-breaking phase transition. After that, some extensions were analyzed for continuous and discrete opinions. For example, the inclusion of competitive interactions [8], three-agents' interactions [12], dynamic self-confidence [16], presence of inflexible agents [17], and others, similarly to was done previously in other opinion dynamics, like the Galam's models [18,19]. In all these extensions the critical behavior of the system was extensively analyzed.

Dynamics of decision-making has been treated in several works in Psychology [20,21] and Neuroscience [22–24]. For the dynamics of opinion formation, we find many models by physicists dedicated to explain the decision-making process or the exchange of opinion through interactions among agents [4]. The mechanisms consider kinetic exchanges (KEOM [7–9,12]), imitation (voter model [25], Sznajd model [26]) or the power of local majorities (majority-rule model [18], majority-vote model [27]), among others. Nevertheless, the inclusion of noise and disorder can be considered in such models [1–4].

Usually discrete opinion models consider two distinct positions or opinions $o = \pm 1$ (yes or no, democrat or republican, candidate A or candidate B). They can be enriched with the inclusion of a third state, $o = 0$, representing neutral state or indecision. Indecision is a current and rising phenomenon which affects both recent and consolidated democracies [28]. Many reasons can lead an individual to become neutral or undecided, for example it can be associated to an anticonformism/nonconformism to the proposals on both sides of the debate. The impact of indecision/neutrality was considered recently in many works [8,12,17,28–33].

In this work we consider a discrete KEOM in the presence of noise and disorder. In addition to pairwise random interactions, we introduce an indecision noise that significantly affects the dynamics of the system. Our aim is to analyze the critical behavior of the model. In this case, based on analytical and numerical results, we found three distinct phase transitions, namely the usual ferro–paramagnetic transition, and two distinct transitions to an

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absorbing state: from the ferromagnetic state and from the paramagnetic one.

2. Model and results

We considered a KEOM [7,8,12] with competitive positive/negative interactions. Our artificial society is represented by N individuals in a fully-connected graph. Each agent i can be in one of three possible opinions at each time step t, i.e., $o_i(t) = +1, -1$ or 0. This general scheme can represent a public debate with two distinct choices, for example *yes* and *no*, and also including the undecided/neutral state. The following microscopic rules control our model:

(1) we choose two agents at random, say i and j, in a way that j will try to persuade i;
(2) with probability $1-q$, the opinion of agent i in the next step $t+1$ is updated according to the kinetic rule $o_i(t+1) = \text{sgn}[o_i(t) + \mu_{ij} o_j(t)]$;
(3) with probability q, the agent i spontaneously change to the neutral state, i.e., $o_i(t+1) = 0$.

In the above dynamic rule, $\text{sgn}(x)$ is the signal function defined such that $\text{sgn}(0) = 0$. This is usual in KEOM, in order to keep all the agents' opinions in one the three possible ones, $+1$, -1 or 0 [8,12,17]. The pairwise couplings μ_{ij} are quenched random variables[1] that follow the discrete probability distribution $F(\mu_{ij}) = p\,\delta(\mu_{ij}+1) + (1-p)\,\delta(\mu_{ij}-1)$. In other words, the parameter p stands for the fraction of negative interactions. As discussed in previous works [8,17], the consideration of such negative interactions produces an effect similar to the introduction of Galam's contrarians in the population [18,34]. In addition, competitive interactions were also considered for the modeling of coalition forming [35]. The probability q acts as a noise in the system, and it allows an autonomous decision of an individual to become neutral [5,36]. It can be viewed as the volatility of some individuals, who tend to spontaneously change their choices. In a two-candidate election, if a given individual does not agree with the arguments of supporters of both sides, he/she can decide to not vote for any candidate, and in this case he/she becomes neutral. In this case, this indecision noise must be differentiated/disassociated from other usual kinds of noises because, unlike the others, it privileges only the neutral opinion. As a recent example, in the 2012 USA election Barack Obama and Mitt Romney disputed for the election for president as the main candidates. It was reported that two months out from election day, nearly a quarter of all registered voters are either undecided about the presidential race or iffy in their support for a candidate, as indicated by polls [37].

For $q = 0$, i.e., in the absence of noise, the model undergoes a nonequilibrium order–disorder (or ferro–paramagnetic) transition at a critical fraction $p_c = 1/4$ [8]. In the ordered ferromagnetic phase, one of the extreme opinions $+1$ or -1 dominates the population, whereas in the disordered paramagnetic phase the three opinions coexist with equal fractions (1/3).

At this point, some definitions are necessary. The order parameter of the system can be defined as

$$O = \left\langle \frac{1}{N} \left| \sum_{i=1}^{N} o_i \right| \right\rangle , \tag{1}$$

that is the "magnetization per spin" of the system, and $\langle \dots \rangle$ stands for average over disorder or configurations, computed at the steady states. Let us also define f_1, f_{-1} and f_0 as the stationary fractions or densities of opinions $+1$, -1 and 0, respectively.

One can start considering the probabilities that contribute to increase and decrease the order parameter. Following [8,12], one can obtain the master equation for O,

$$\begin{aligned}\frac{d}{dt} O = {} & q f_{-1} + (1-q)[(1-p) f_1 f_{-1} + p f_{-1}^2 \\ & + (1-p) f_0 f_1 + p f_0 f_{-1}] \\ & - q f_1 - (1-q)[(1-p) f_1 f_{-1} + p f_1^2 + (1-p) f_0 f_{-1} \\ & + p f_0 f_1] = 0 \, .\end{aligned} \tag{2}$$

In the stationary state $dO/dt = 0$. Using the normalization condition $f_1 + f_{-1} + f_0 = 1$, we obtain two solutions for Eq. (2) in the stationary state, namely $2f_1 + f_0 = 1$, which implies in $f_1 = f_{-1} = (1-f_0)/2$ (disordered solution), or

$$f_0 = \frac{q + p(1-q)}{(1-p)(1-q)} . \tag{3}$$

In this case, Eq. (3) is valid in the ferromagnetic phase. We emphasize that $q = 0$ leads to $f_0 = p/(1-p)$, which agrees with the result of Ref. [8]. One can obtain another equation for f_0 considering the fluxes into and out of the neutral state $o = 0$. In this case, the master equation for f_0 is given by

$$\begin{aligned}\frac{d}{dt} f_0 = {} & q(f_1 + f_{-1}) + p(1-q)(f_1^2 + f_{-1}^2) \\ & + 2(1-p)(1-q) f_1 f_{-1} \\ & - (1-p)(1-q) f_0 (f_1 + f_{-1}) \\ & - p(1-q) f_0 (f_1 + f_{-1}) \, .\end{aligned} \tag{4}$$

Considering the disordered phase, where $f_1 = f_{-1} = (1-f_0)/2$, Eq. (4) gives us in the stationary state (where $df_0/dt = 0$)

$$(1-q)\left(\frac{1-f_0}{2}\right)^2 = [(1-q) f_0 - q]\left(\frac{1-f_0}{2}\right) , \tag{5}$$

which gives us two solutions, namely $f_0 = 1$ which can be ignored by considering the steady state of the other two fractions f_1 and f_{-1} [8], or

$$f_0 = \frac{1+q}{3(1-q)} . \tag{6}$$

In this case, Eq. (6) is valid in the paramagnetic phase. The above equations (3) and (6) are both valid at the critical point, and we can equate them to obtain

$$q_c(p) = \frac{1-4p}{2(1-p)} . \tag{7}$$

These critical noises separate the ferromagnetic and the paramagnetic phases. As discussed above, in the ferromagnetic phase one of the extreme opinions $+1$ or -1 dominates the population (one of the sides wins the debate), whereas in the paramagnetic phase the two extreme opinions coexist ($f_1 = f_{-1}$, i.e., there is no decision). Notice that we recover $f_0 = 1/3$ in Eq. (6) and $p_c = 1/4$ in Eq. (7) for $q = 0$, in agreement with [8].

In order to obtain an analytical expression for the order parameter, one can consider the fluxes into and out of the state $o = +1$.

[1] The nature of the random variables μ_{ij} does not affect our results, they can also be considered as annealed variables.