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Quench dynamics of two coupled zig-zag ion chains

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ABSTRACT

We explore the non-equilibrium dynamics of two coupled zig-zag chains of trapped ions in a double well potential. Following a quench of the potential barrier between both wells, the induced coupling between both chains due to the long-range interaction of the ions leads to the complete loss of order in the radial direction. The resulting dynamics is however not exclusively irregular but leads to phases of motion during which various ordered structures appear with ions arranged in arcs, lines and crosses. We quantify the emerging order by introducing a suitable measure and complement our analysis of the ion dynamics using a normal mode analysis showing a decisive population transfer between only a few distinguished modes.

1. Introduction

Cooled ions in traps form a clean and highly versatile setup for exploring structure formation with long-range interacting particles, both in equilibrium and non-equilibrium. In equilibrium, possible structures include small and large ion crystals [1–3] possessing various internal ordering such as concentric rings (2D), shells (3D) [4–6] and string-of-disks configurations [7], and even two-component Coulomb bicrystals [8]. At the crossover from one to higher dimensions trapped ions can also form zig-zag configurations, a structure that attracts particular attention in the recent literature [9–11]. Specifically, varying the geometry of an anisotropic harmonic trap allows for a second-order phase transition from a linear to a zig-zag structure, which can either occur without topological defects [12,13] or involve topological defects so-called kinks [14–16]. Out of equilibrium, the recent literature predicts an equally rich variety of possible ionic structures including defects in Coulomb crystals [17–21] spatiotemporal patterns in laser-driven microtraps [22] and periodic lattices [23,24], but also interaction induced current reversals of the transport direction [25] based on structure formation in the phase space. Much of this research on structure formation with trapped ions roots in the admirable advancements of the controllability of ions in recent years. This ranges from the quickly progressing miniaturization of ion traps and lab on chip technologies [26,27] via the advent of optical trapping techniques [28] to the discovery of multi-segmented Paul and Penning traps [29,30]. The latter example in particular allows for more and more complex but still controllable arrangements of long-range interacting particles as required e.g. for quantum information processing [31–34]. The above advancements allow and evoke a new type of question: How do individual ionic structures respond if we couple them to each other? Consider for example a segmented ion trap with two wells, both loaded with an individual zig-zag configuration, separated from each other by a potential barrier between the wells. Let us now quench the barrier to a lower value, which increases the coupling between the individual chains: Are the only two (expected) alternatives for the dynamics of the zig-zag chains that they either deform only slightly and respond with small oscillations to the increased coupling or that we observe their complete melting resulting in irregular oscillations of all ions? This is precisely the problem we want to investigate in the present work. To explore the above problem we develop a minimal model based on a two dimensional double well potential that allows for zig-zag configurations in both wells whose geometries resemble the well-known zig-zag states in anisotropic single well traps. Remarkably, following the quench of the barrier we observe that the restructuring process does not simply lead to irregular oscillations but to a complex non-equilibrium dynamics constituted of different phases of motion. Phases of irregular oscillatory motion are interrupted by motional phases which exhibit transient ordered configurations. Although nonlinear dynamics governs the motion of the coupled ion chains, we employ a normal mode analysis showing that the population of the corresponding linear eigenvectors is not arbitrarily distributed over the whole band of modes as one would expect for e.g. a chaotic system (see [35] and references therein for chaotic behavior in trapped ion clusters). Instead, during most phases of the time evolution only a few eigenvectors

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are strongly populated and nonlinear effects show up in form of a decisive and quite sudden transfer of energy among the different modes. Our work is organized as follows. Section 2 explains our setup and the preparation of the ground state configuration. Section 3 provides our main results followed by a normal mode and population analysis of the dynamics. We summarize our findings and their interpretation in section 4.

2. Setup, Hamiltonian and ground state configuration

We consider N ions, described as classical point particles with mass m and charge Q, confined in radial direction (x, y) to a linear quadrupole Paul trap and to a double well potential (segmented trap) in z-direction.[1]

$$\Phi(x, y, t) = \frac{U_{\mathrm{dc}}}{2}(cx^2 + cy^2) + \frac{U_{rf}}{2}\cos(\omega_{\mathrm{rf}}t)(cx^2 - cy^2), \qquad (1)$$

with U_{dc} and U_{rf} being the applied constant and the rf-voltage; ω_{rf} is the (radio)frequency, c is a geometrical parameter of the trap. The geometrical parameter is for both directions x and y equal in radially symmetric traps but in planar traps [39,30] the geometrical parameter for both directions can strongly differ by the design of the traps. The ion dynamics in the radio-frequency trap is composed of the so-called micro motion, and a comparatively slow averaged motion taking place in an effective harmonic potential [40] $V(x, y) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2)$. Here, $\omega_x = \frac{\omega_{rf}}{2}\sqrt{a - q^2/2}$ and $\omega_y = \frac{\omega_{rf}}{2}\sqrt{a + q^2/2}$ are the effective trapping frequencies with $a = \frac{4QU_{dc}}{m\omega_{rf}^2}c$ and $q = \frac{2QU_{rf}}{m\omega_{rf}^2}c$ being dimensionless parameters. For the confinement in z-direction we assume the following phenomenological double well potential [41], with wells centered at $\approx \pm z_0$ and separated from each other by a barrier of height $\sim 1/C$ (see Fig. 1).

$$V_{\mathrm{d}}(z) = \frac{m}{2}\omega_z^2 z_0^2 + \frac{m}{2}\omega_z^2 z^2 - \frac{m}{2}\sqrt{4C^2 + 4\omega_z^4 z^2 z_0^2} \qquad (2)$$

This potential quantitatively resembles the shape of individual harmonic wells around $\pm z_0$ up to terms proportional to C^2.

Specifically, for a given sufficiently high barrier this allows us to prepare zig-zag chains in each of the two wells which are the energetically lowest equilibrium configuration of the double well which we call the ground state configuration in the following. Note that finding the many-particle minimum of a many ion system is generally a highly nontrivial task; hence the present choice of the double well potential is a crucial step to allow for a numerical study of the dynamics of coupled ion chains. After preparing this configuration, i.e. its numerical determination, our strategy will be to ramp down the barrier height by a certain amount which corresponds to a quench of the quantity C. Subsequently the resulting dynamics of the now strongly coupled ion chains will be explored. To understand the complex dynamics of coupled many-ion structures in non-equilibrium it is crucial to simplify our model. First, since we are interested in the dynamics on large scales we neglect the micromotion. Second, we focus on a two-dimensional description, which simplifies the visualization of the ionic structures and their analysis but does not change qualitatively the resulting dynamics and phenomenology of the structure forming processes.

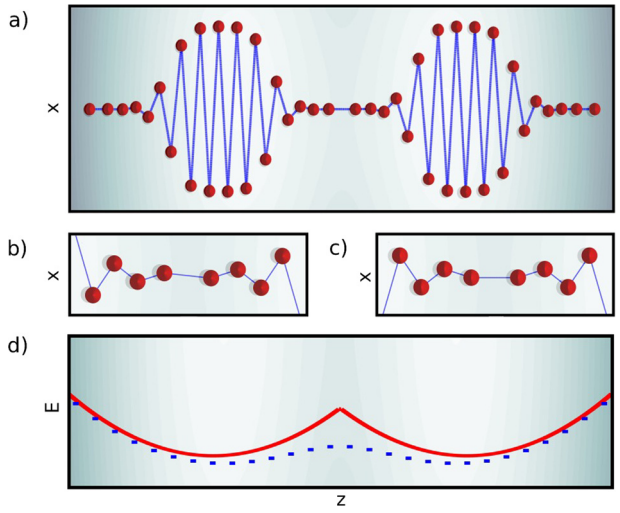

Fig. 1. (a) Cartoon of the double zig-zag equilibrium configuration in the double well potential used as the initial configuration, calculated for the used parameters. (b, c) Magnifications of the part of the ionic configuration which links between the two chains, i.e. in the barrier region. It highlights the difference between the ground state configuration (b) and the energetically next higher equilibrium configuration (c) which we call the mirror configuration. (d) The red (solid) line shows the double well potential before the quench and the blue (dashed) line after the quench. A lowered barrier enhances the coupling between the two ion chains (arbitrary units throughout).

Specifically, we choose parameters $\alpha = \omega_x/\omega_z \approx 8.3$ where the ground state configuration is a planar (2D) zig-zag structure in the x–z-plane far from the transition to a helical (3D) zig-zag chain. The transition from the 2D structure to the 3D helical zig-zag chain occurs at $\alpha \approx 4.5$, whereas the transition to a linear 1D line-structure occurs at $\alpha \approx 10$ (for $\omega_x = \omega_y$).[2] Thus, although ramping the barrier will generally produce both in-plane and out-of-plane fluctuations, the latter ones are generically small. A strong confinement in y-direction ($\omega_y/\omega_z \geq 10$) prevents their amplification in the course of the consecutive dynamics; hence the ions stay close to the x–z plane. Indeed the impact of small out-of plane fluctuations on the intra-plane dynamics is quadratically suppressed with the distance of the ions perpendicular to the x–z plane[3] and consequently a projection of the ion dynamics in 3D looks very similar to a direct 2D description. Hence, we focus on a two-dimensional minimal model given by the two-dimensional Hamiltonian:

$$H(\{\mathbf{r}_i, \mathbf{p}_i\}) = \sum_{i=1}^{n} \frac{\mathbf{p}_i^2}{2m} + \sum_{i=1}^{n}[V_d(z) + V(x)] + \sum_{i=1, j<i}^{n} \frac{Q^2}{4\pi\epsilon_0 r_{ij}} \qquad (3)$$

with $V(x) = m\omega^2 x^2/2$, $\mathbf{r}_i = (x_i, z_i)$, $r_{ij} = \sqrt{(x_i - x_j)^2 + (z_i - z_j)^2}$ and $\mathbf{p} = (p_x, p_z)$. Introducing a rescaled time $t_u = 1/\omega_z$ and space units $x_u = K \equiv [Q^2/(4\pi\epsilon_0 m\omega_z^2)]^{1/3}$ and defining

[1] Note that formally, the Laplace equation does not allow to combine the linear quadrupole trap potential, which we choose for simplicity, with a double well potential in axial direction. A complex form of the dc part in the radial direction is necessary to compensate the double well term in the axial direction. However, recent experiments with segmented Paul traps (e.g. [36–38]) could indeed realize the combination of a linear Paul trap, similar to the potential we have chosen here, and a Mexican hat like potential which justifies to consider a combination of a double well potential with a linear Paul trap.

[2] These values for the aspect ratios have been calculated for the used double well potential and differ slightly from the values in an anisotropic harmonic potential [9,42].

[3] The only force which couples the dynamics of ions perpendicular to the x–z plane to the intra-plane dynamics is the Coulomb coupling. Consider two ions in distance $L = \sqrt{L_\parallel^2 + L_\perp^2}$ where $L_\parallel$ is the projection of their distance onto the x–z plane and $L_\perp$ is their distance perpendicular to this plane. Then $F_\parallel = F\cos[\tan(L_\perp/L_\parallel)] \approx F[1 - (L_\perp/L_\parallel)^2/2]$ for $L_\parallel \gg L_\perp$, showing that forces produced by small out-of plane oscillations onto the intra-plane dynamics are quadratically suppressed.