Physics Letters A 380 (2016) 1231–1235

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Intrinsic gravitomagnetism and non-commutative effects

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ARTICLE INFO

Article history:
Received 5 October 2015
Received in revised form 30 January 2016
Accepted 1 February 2016
Available online 3 February 2016
Communicated by P.R. Holland

Keywords:
Stationary gravito-magnetic fields
Non-commutative space

ABSTRACT

We give a demonstration that gravito-magnetism and non-commutative geometry predict an orbit precession for a test particle moving in the central force obeying the inverse-square law. To this end, the two equations of motion for a test particle are compared with each other, one in the non-commutative space involving a static solid sphere and the other in the usual space involving a slowly rotating sphere. The comparison indicates that the non-commutative effect is similar to the intrinsic gravitomagnetic contribution to the orbit precession.

1. Introduction

In the context of general relativity and gravitational interactions, the study of analogies (and differences) between gravitation and classical electrodynamics has an area of interest and is dated back to the second half of the nineteenth century. Many of the differences are due to the tensorial nature of the gravitational interaction and the nonlinearity of the Einstein field equations.

The mass current produces a field called Gravitomagnetic (**GM**) field and gravitomagnetism emerges as a consequence of mass currents, analogous to the generation of magnetic field by electric (charge) current.

The two main types of mass currents which are encountered in gravitomagnetism are:

1) *Translational motion*: It generates an *Extrinsic* GM field that depends on the frame of reference of observer and hence it can be eliminated in the rest frame of the matter.

2) *Rotational motion around the body's center of mass*: It generates an *Intrinsic* GM field and is directly related to the proper body's angular momentum (spin). This field is not generable and eliminable with any Lorentz transformation or any other frame and coordinate transformation. Many of the researches in gravitomagnetism have focused on the discussion of various properties of this type. To give a comprehensive review of various aspects of the intrinsic gravitomagnetism and depth study, the reader may refer to worthwhile textbooks as [1,2].

An important aspect of the intrinsic gravitomagnetism is related to its influence on orbit of a test particle about a rotating central mass. That means, the gravitomagnetic interaction plays a part in shaping the orbits in a rotating central force field. In this regard, one of the most important effects is to produce the orbit precession and the followings are the important cases:

(i) the precession of a gyroscope in orbit,

(ii) the precession of orbital planes in which a mass orbiting a large rotating body constitutes a gyroscopic system whose orbital axis will precess,

(iii) the precession of the pericenter of the orbit of a test mass about a massive rotating object.

The first and second cases are due to the one particularly interesting aspect of gravity is the so-called Thirring–Lense or Frame-Dragging effect: A rotating mass should drag space–time around it, producing a detectable gyroscopic precession, or affecting for example the orbit of satellites around the Earth. The *Gravity Probe B experiment* and the *LAGEOS satellites* have aimed to measure these effects [3].

In this article, within the context of case (iii), we consider a slowly rotating homogeneous sphere whose gravitational (gravito-electric and magnetic) fields are well known [4]. As we know, when a particle is acted on by an (attractive) inverse-square force comes from a central source (say a static spherical object), the trajectory of particle is a Keplerian orbit. In the case of the source rotation, the orbit undergoes a precession. A discussion and illustration of the precession of this type can be found in [5].

On the other hand, the Noncommutative (**NC**) geometry has played an increasingly important role, more notably, in the attempts to understand the space–time structure at very small distances. The NC structure idea at small length scale first introduced by Snyder [6]. He applied the NC structure concept for discrete space–time coordinates instead of the continuum ones.

In more recent times a great deal of interest has been generated in this area of research. The NC geometry provides valuable tools

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http://dx.doi.org/10.1016/j.physleta.2016.02.005
0375-9601/

for the study of physical theories in classical and quantum level. In recently decades, the NC version of physical theories has been widely employed (in many of the research works). The main motivations that drew attention to NC field theories came from string theory and quantum gravity [7].

The NC theories may also be justified in their own right, because of the interesting predictions they have made in the physical theories on the real world. For instance, they are employed to explain the Quantum Hall effect [8], Landau problem [9], or their applications to particle physics [10]. Other examples of applications are the IR/UV mixing and non-locality [11], Lorentz violation and new physics at Planck scales [12,13].

Moreover, an special attention has been paid for the NC version and extension of cosmological scenarios in classical and quantum perspective. A few examples of resulting NC classical and quantum cosmology of such models have been studied in different works [14].

Also, in the latest decades, several investigations have been carried out to clarify the possible role of noncommutativity in the gravitational systems as black holes and the other related areas in Classical Mechanics. As an important example, the NC effects in central force orbits are studied in [15–17] and we will be concerned with this case in the present work.

The NC concept is not limited only to the space–time coordinates or operators, and can be extended to the phase space classical variables as well. In a general phase space, the noncommutativity is described by the NC parameters α_{ij} (*corresponding to the position coordinates sector*) and β_{ij} (*corresponding to the momentum coordinates sector*). Hence, the Hamilton motion equations contain terms including the NC parameters which appear as additional force terms with respect to the usual space. Also, when in the usual space, an electromagnetic field is present, the Lorentzian force gives the motion equation of a moving charged particle. Under special conditions, by comparing these motion equations (in the NC and usual spaces), one can deduce that the effect of the NC parameters on the motion equation is equivalent to the effect of the magnetic field in the usual space. This equivalence has been demonstrated in the classical and in the quantum perspectives, e.g. [18].

Analogy between gravitation (in the weak field limit) and electromagnetism permits the extension of such equivalence between the NC effects and GM field. In the previous work [19], the equivalence between

NC parameter β_{ij} and extrinsic (constant) GM field,

has been studied and in the current work, the possibility of equivalence between

NC parameter α_{ij} and intrinsic (variable) GM field,

is investigated.

It should be noted that the case of extrinsic (*constant*) GM field is somewhat speculative (*gedanken*), that is, it may has not an analogous in the real world. While the case of intrinsic GM fields have real analogous in the world around us, for example, strong gravitational fields produced by rotating black holes and the other spinning celestial massive objects. From this point of view, the present work may have relevance.

The work is organized as follows:

In section 2, a brief review of the basic features of NC classical mechanics is given. In section 3, we introduce the equations of motion for a test particle moving in the NC and usual spaces. In section 4, the similarity between the NC effects and the intrinsic GM field is presented. A summary of the work is given in the last section.

2. Noncommutative classical mechanics

From the quantum theory point of view, the coordinates operators of NC space–time do not satisfy the usual commutation relations, that is $[\hat{x}_i, \hat{x}_j] = 0$ ($\hat{x}_i$ denote coordinates operators), and instead obey non-trivial ones. The commutation relations are defined with respect to the notion of $*$ (or NC product) such that for any two different operators, say $\hat{A}$ and $\hat{B}$, one has, in general $\hat{A} * \hat{B} \neq \hat{B} * \hat{A}$. Usual quantum mechanics is formulated on commutative spaces satisfying the following commutation relations:

$$[\hat{x}_i, \hat{x}_j] = 0, \quad [\hat{p}_i, \hat{p}_j] = 0, \quad [\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}, \tag{1}$$

and NC quantum mechanics satisfies the non-commutative relations:

$$[\hat{x}_i, \hat{x}_j] = i\hbar\alpha_{ij}, \quad [\hat{p}_i, \hat{p}_j] = 0, \quad [\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}, \tag{2}$$

where the NC parameter α_{ij} is real and anti-symmetric c-number.[1] As it can be easily seen, $\hbar\alpha_{ij}$ must have dimension of *Area* and in consequence α_{ij} has dimension of *Time/Mass*.

Transition from quantum mechanics to classical mechanics is facilitated by the *Dirac Quantization Condition*:

$$\frac{1}{i\hbar}[\hat{A}, \hat{B}] \to \{A, B\} \tag{3}$$

where A, B are two functions of canonical variables corresponding to two quantum operators $\hat{A}, \hat{B}$ and $\{A, B\}$ denotes their Poisson Bracket.

So, in analogy with the commutation rules (2), the NC classical mechanics can be constructed as

$$\{x_i, x_j\} = \alpha_{ij}, \quad \{x_i, p_j\} = \delta_{ij}, \quad \{p_i, p_j\} = 0. \tag{4}$$

If the Hamiltonian is given by

$$H = \frac{p_i p^i}{2m} + V(x_i), \tag{5}$$

it is easily to show that the Hamilton's equations corresponding to the NC structure (4) are

$$\dot{x}_i = \frac{p_i}{m} + \alpha_{ij}\frac{\partial V}{\partial x^j}, \tag{6}$$

$$\dot{p}_i = -\frac{\partial V}{\partial x^i}. \tag{7}$$

The last equations will be used to obtain the motion equations of a test particle moving in the central potential $V = V(r)$.

3. Equations of motion

In the two subsections below, we give the equations of motion for the two following systems:

1) A test particle moving in the NC space acted upon by a central force field generated by (static) rigid sphere,

2) A test particle moving in the usual space acted upon by the stationary gravito-electromagnetic field generated by a slowly rotating homogeneous sphere.

[1] The anti-symmetric property is usually defined in terms of Levi-Civita symbol ϵ_{ijk}, that is $\alpha_{ij} = \epsilon_{ijk}\alpha_k$ and hence α_k has the same nature of α_{ij}.