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Electron scattering in a graphene nanoribbon in the presence of ferromagnetic layer and Rashba interaction

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ABSTRACT

We study the possibility to control the spin polarization and spin-dependent transport in a graphene sheet by considering a ferromagnetic layer in the presence of the Rashba spin–orbit interaction. Studying the scattering problem with the help of the Green function (which was found explicitly), we obtained simple analytical expressions for the spin dependent transmission probability. Using the small exchange parameter and Rashba coupling constant, we can obtain any degree of spin polarization, but in the case of a small interaction region, only for slow electrons.

1. Introduction

The transport properties of the graphene continue to attract a lot of attention due to the possible applications in nanoelectronic devices [1,2]. These applications require the control of the spin current, often using the exchange interaction and Rashba spin–orbit coupling [3–6].

In this paper, we study with mathematical rigor the possibility to control the spin polarization and spin-dependent transport in a graphene sheet by considering a ferromagnetic substrat (or cover layer) in the presence of the Rashba spin–orbit interaction [7]. As a result, the exchange field can be induced in graphene due to the magnetic proximity effect [8]. The Rashba coupling can arise from symmetry breaking generated by a ferromagnetic layer or by an external electric field.

We consider two extreme cases, when the region with a ferromagnetic layer and Rashba interaction is infinitely extended and when it is sufficiently localized. It turns out that, as in other quasi-one-dimensional systems [9,10], the small interactions can in both cases significantly affect the scattering and polarization of electrons. Considering the scattering problem, in some cases we obtained simple analytical expressions for the spin dependent transmission probability. Since the conductance is approximately proportional to the transmission probability [11], in fact we are dealing with the conductance.

It has been found that in the case of the sufficiently large interaction region, for small Fermi energy, μ, and α where μ is the exchange field parameter, and α is the Rashba coupling strength, theoretically we can obtain any degree of spin polarization by the choice of the value μ/α, and the conductance does not depend on the presence of an impurity. We note that the spin polarization increases with increasing μ/α. If the Fermi energy is high enough, a large polarization can be obtained for a small impurity potential and slow electrons. In the case of the small interaction region, we can achieve a significant degree of polarization of initially unpolarized current only for slow electrons and if the Fermi energy is large enough.

2. Green function in the case of constant α and μ

We use the Hamiltonian $H = H_0 + H_R + H_\mu$. Here H_0 for both spin components in the one-valley approximation is given by [12]

$$H_0 = \hbar v_F (\sigma_1 \partial/\partial x + \sigma_2 \partial/\partial y)$$

where $\hbar$ is the Planck constant, $v_F > 0$ is the Fermi velocity, and σ_j, $j = 1, 2$ are the Pauli matrices acting on the pseudospin components [12]; further without loss of generality, we consider $\hbar v_F = 1$. This term is the Dirac Hamiltonian for massless fermions. The Rashba term reads [7] $H_R = \alpha(\sigma_2 s_1 - \sigma_1 s_2)$; here $\alpha = \text{const}$ is the strength of the Rashba spin–orbit coupling and s_j, $j = 1, 2, 3$

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are the Pauli matrices acting in the spin space. The effect of the ferromagnetic layer is described by the exchange Hamiltonian $H_\mu = \mu s_z$ [6,7] where μ is the effective exchange field. Hamiltonian H acts on the functions

$$\psi(x, y) = (\psi_1(x, y), \psi_2(x, y), \psi_1'(x, y), \psi_2'(x, y)) \tag{1}$$

with the domain $-\infty < x < \infty$, $0 \le y \le l$ satisfying periodic boundary conditions $\psi(x, 0) = \psi(x, l)$. This means neglecting the boundary effects, and l is assumed to be large enough. The index $j = 1, 2$ in (1) indicates the pseudospin components; the components ψ_1, ψ_2 (ψ_1', ψ_2') correspond to the spin-up (spin-down, respectively) electrons. We note that similar Hamiltonians are used in the study of the anomalous Hall effect [6].

In this section, we suppose that $\alpha, \mu = const$; it corresponds to the great length as the ferromagnetic layer, and the region of the Rashba interaction. To study the scattering problem by the Lippmann–Schwinger equation, we need the Green function of H. We will find the Green operator $(H - E)^{-1}$ by solving the equation

$$(H - E)\psi = \varphi \tag{2}$$

with respect to ψ. Expanding $\psi = \psi(x, y)$ in the basis $(1/\sqrt{l}) \times \exp(-2\pi i n y/l)$, $n = 0, \pm1, \pm2, \dots$ at fixed x, we obtain

$$\psi_j^{(')}(x, y) = \frac{1}{\sqrt{l}} \sum_{n=-\infty}^{\infty} \psi_{jn}^{(')}(x) \exp(-2\pi i n y/l), \quad j = 1, 2. \tag{3}$$

Using (1), (3), we write (2) as the set of independent linear systems of the form

$$\begin{aligned}
&-i d\psi_{2n}/dx + (2\pi i n/l)\psi_{2n} - (E - \mu)\psi_{1n} = \varphi_{1n},\\
&-i d\psi_{1n}/dx - (2\pi i n/l)\psi_{1n} + 2i\alpha\psi_{1n}' - (E - \mu)\psi_{2n} = \varphi_{2n},\\
&-i d\psi_{2n}'/dx + (2\pi i n/l)\psi_{2n}' - 2i\alpha\psi_{2n} - (E + \mu)\psi_{1n}' = \varphi_{1n}',\\
&-i d\psi_{1n}'/dx - (2\pi i n/l)\psi_{1n}' - (E + \mu)\psi_{2n}' = \varphi_{2n}',
\end{aligned} \tag{4}$$

$n = 0, \pm1, \pm2, \dots$. Thus, H can be considered as the operator acting on functions $\psi_n(x) = (\psi_{1n}(x), \psi_{2n}(x), \psi_{1n}'(x), \psi_{2n}'(x))$ at the fixed n. In this case, we will talk about the energy spectrum or the scattering in the nth subband.

After Fourier transform $\widehat{\psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-ikx}\psi(x)dx$, we obtain from (4) the system

$$\begin{aligned}
&(k + 2\pi i n/l)\widehat{\psi}_{2n} - (E - \mu)\widehat{\psi}_{1n} = \widehat{\varphi}_{1n},\\
&(k - 2\pi i n/l)\widehat{\psi}_{1n} + 2i\alpha\widehat{\psi}_{1n}' - (E - \mu)\widehat{\psi}_{2n} = \widehat{\varphi}_{2n},\\
&(k + 2\pi i n/l)\widehat{\psi}_{2n}' - 2i\alpha\widehat{\psi}_{2n} - (E + \mu)\widehat{\psi}_{1n} = \widehat{\varphi}_{1n}',\\
&(k - 2\pi i n/l)\widehat{\psi}_{1n}' - (E + \mu)\widehat{\psi}_{2n}' = \widehat{\varphi}_{2n}',
\end{aligned} \tag{5}$$

$n = 0, \pm1, \pm2, \dots$. The determinant of this system is equal to

$$\begin{aligned}
\Delta_n(E, k) = {} & [(E - \mu)^2 - (k^2 + (2\pi n/l)^2)]\\
& \times [(E + \mu)^2 - (k^2 + (2\pi n/l)^2)]\\
& - 4\alpha^2(E^2 - \mu^2).
\end{aligned} \tag{6}$$

It is obvious that the energy subbands decrease with increasing $|n|$, therefore, the spectrum of H coincides with the spectrum in the 0th subband and is the union of $\left(-\infty, -|\alpha\mu|/\sqrt{\alpha^2 + \mu^2}\right)$ and $\left(|\alpha\mu|/\sqrt{\alpha^2 + \mu^2}, \infty\right)$. If $\alpha, \mu \ne 0$, then there is a gap in the spectrum.

To find the Green function of the Hamiltonian H, it is sufficient to solve the system (5) for an arbitrary n. We introduce the notation

$$k_n^{\pm} = \sqrt{E^2 - (2\pi n/l)^2 + \mu^2 \pm 2\sqrt{E^2(\alpha^2 + \mu^2) - \alpha^2\mu^2}}, \tag{7}$$

then, by (6),

$$\frac{1}{\Delta_n(E, k)} = \frac{1}{(k_n^+)^2 - (k_n^-)^2}\left(\frac{1}{(k_n^-)^2 - k^2} - \frac{1}{(k_n^+)^2 - k^2}\right). \tag{8}$$

Using (8), Cramer's rule, and known formula $\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \frac{e^{ikx}\widehat{\varphi}(k)}{k^2 - a^2} dk = -\frac{1}{2ia}\int_{\mathbb{R}} e^{ia|x - x'|}\varphi(x')dx'$, we can derive the Green function of H in the coordinate representation:

$$\begin{aligned}
\psi_{1n}(x) &= ((H - E)^{-1}\varphi)_{1n}(x)\\
&= \frac{1}{2ik_n^+} \int_{\mathbb{R}} e^{k_n^+|x - x'|}\Big(-(E - \mu)\varphi_{1n}(x')\\
&\quad - \big(k_n^+ \operatorname{sgn}(x - x') + 2\pi i n/l\big)\varphi_{2n}(x')\\
&\quad + 2i\alpha\varphi_{2n}'(x') - \frac{1}{(k_n^+)^2 - (k_n^-)^2}\\
&\quad \times \Big(-2\big(E\mu + \sqrt{E^2(\alpha^2 + \mu^2) - \alpha^2\mu^2}\big)\big((E - \mu)\varphi_{1n}(x')\\
&\quad + \big(k_n^+ \operatorname{sgn}(x - x') + 2\pi i n/l\big)\varphi_{2n}(x')\big)\\
&\quad + 4\alpha^2(E + \mu)\varphi_{1n}(x')\\
&\quad - 2i\alpha\big(k_n^+ \operatorname{sgn}(x - x') + 2\pi i n/l\big)(E + \mu)\varphi_{1n}'(x')\\
&\quad - 2i\alpha\big(E^2 + \mu^2 - 2\sqrt{E^2(\alpha^2 + \mu^2) - \alpha^2\mu^2}\\
&\quad - 2(2\pi n/l)^2 + (4\pi i n/l)k_n^+ \operatorname{sgn}(x - x')\big)\varphi_{2n}'(x')\Big)\Big)dx'\\
&\quad + \frac{1}{2ik_n^-((k_n^+)^2 - (k_n^-)^2)} \int_{\mathbb{R}} e^{k_n^-|x - x'|}\Big(-2\big(E\mu\\
&\quad + \sqrt{E^2(\alpha^2 + \mu^2) - \alpha^2\mu^2}\big)\big((E - \mu)\varphi_{1n}(x')\\
&\quad + \big(k_n^- \operatorname{sgn}(x - x') + 2\pi i n/l\big)\varphi_{2n}(x')\big)\\
&\quad + 4\alpha^2(E + \mu)\varphi_{1n}(x')\\
&\quad - 2i\alpha\big(k_n^- \operatorname{sgn}(x - x') + 2\pi i n/l\big)(E + \mu)\varphi_{1n}'(x')\\
&\quad - 2i\alpha\big(E^2 + \mu^2 - 2\sqrt{E^2(\alpha^2 + \mu^2) - \alpha^2\mu^2}\\
&\quad - 2(2\pi n/l)^2 + (4\pi i n/l)k_n^- \operatorname{sgn}(x - x')\big)\varphi_{2n}'(x')\Big)dx';
\end{aligned}$$

$$\begin{aligned}
\psi_{2n}(x) &= ((H - E)^{-1}\varphi)_{2n}(x)\\
&= \frac{1}{2ik_n^+} \int_{\mathbb{R}} e^{k_n^+|x - x'|}\Big(-(E - \mu)\varphi_{2n}(x')\\
&\quad - \big(k_n^+ \operatorname{sgn}(x - x') - 2\pi i n/l\big)\varphi_{1n}(x')\\
&\quad + \frac{1}{(k_n^+)^2 - (k_n^-)^2}\Big(2\big(E\mu + \sqrt{E^2(\alpha^2 + \mu^2) - \alpha^2\mu^2}\big)\\
&\quad \times \big((E - \mu)\varphi_{2n}(x') + \big(k_n^+ \operatorname{sgn}(x - x') - 2\pi i n/l\big)\varphi_{1n}(x')\big)\\
&\quad + 2i\alpha(E^2 - \mu^2)\varphi_{1n}'(x')\\
&\quad + 2i\alpha(E - \mu)\big(k_n^+ \operatorname{sgn}(x - x') + 2\pi i n/l\big)\varphi_{2n}'(x')\Big)\Big)dx'\\
&\quad - \frac{1}{2ik_n^-((k_n^+)^2 - (k_n^-)^2)} \int_{\mathbb{R}} e^{k_n^-|x - x'|}\Big(2\big(E\mu\\
&\quad + \sqrt{E^2(\alpha^2 + \mu^2) - \alpha^2\mu^2}\big)\big((E - \mu)\varphi_{2n}(x')
\end{aligned}$$