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Pentamodal property and acoustic band gaps of pentamode metamaterials with different cross-section shapes

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ABSTRACT

Pentamodal property and acoustic band gaps of pentamode metamaterials with different cross-section shapes, including regular triangle, square, pentagon, hexagon and circle, have been comparatively studied by finite-element method. Results show that for the varying diameters of circumcircles in thick and thin ends of unit (D and d), the ratio of bulk modulus to shear modulus (B/G) and bandgaps of these five structures perform similar changing tendency. With the increasing d, B/G decreases and the single-mode bandgap moves toward high-frequency direction with the decreasing normalized bandwidth ($\Delta\omega/\omega_g$). With the increasing D, B/G keeps around the respective average value, and the single-mode bandgap firstly moves to high-frequency then to low-frequency direction with the firstly increasing and then decreasing $\Delta\omega/\omega_g$. Complete bandgap appears as D reaching to critical value for each given d, then moves to high-frequency direction. For same parameters the triangle case has highest B/G and acoustic band gaps with lower frequency and broader bandwidth.

1. Introduction

For the past decades, there have been a great interest in the study of the optical/electromagnetic metamaterials for their remarkable properties and potential applications [1–4]. Recently, the attention has been extended to acoustic/elastic metamaterials [5–9], which perform various applications such as acoustic absorber and cloaking device. Milton et al. [10] and Sigmund [11] independently suggested the "pentamode" from the angle of theoretical analysis in 1995. The pentamode structure performs novel physical property that its bulk modulus B is much larger than shear modulus G, resulting in that B/G is obviously greater than that of natural materials. And then the pentamode structures are easy to deform and difficult to compress for their higher B/G, which means that they can display the fluid-like behaviors. Therefore, pentamode metamaterials are a kind of newly-developing mechanical metamaterial, and they are more attractive for their special mechanical property and the effective control of the propagation of elastic/acoustic wave, which can be used to realize the elastic or acoustic free-space cloaking.

Researches on pentamode metamaterials are gradually carrying out in last years. The recent reports mainly focus on two aspects. One is about the mechanical behavior of pentamode structures, which mostly reported by Kadic, Bückmann and colleagues at the Karlsruhe Institute of Technology (KIT). They constructed the pentamode structures and verified their well mechanical properties in experiments [12,13]. Also they designed the elasto-mechanical cloak using the pentamode structures and realized the elastically hiding of objects [14]. The other is about acoustic cloaks composed of pentamode metamaterials basing on the acoustic cloaking theory from Norris [15]. The special transformation theory has been derived [16] and the theoretical verifications of the feasibility for pentamode acoustic cloaking have been continually presented [17,18].

Most reports on pentamode structures mainly related to the classical structure presented by Milton et al. [12–14,19,20] and the minor structural modifications for the pentamode materials appeared in past two years [21–23]. However, the significant structural innovations for the pentamode materials have been rarely reported, and it is essential to find out the band gap property of pentamode metamaterials for the fact that numerous important applications of metamaterials relies on the existence of wide frequency band gaps. In our previous work, the quantitative characterization of pentamode structures with cubic system has been presented

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basing on the elasto-dynamic theory. The pentamode property includes two necessary factors: the higher B/G and the decouple of compression and shear waves. By analyzing the phonon band structure, the B/G can be calculated from theoretical equations and the decouple can be realized by finding the single-mode band gap.

Therefore, in this paper we studied the pentamodal property and the acoustic band gaps of pentamode metamaterials with different cross-section shapes. The primary unit of pentamode metamaterials is no longer limited to the double-cone element, and its cross-section shape includes regular triangle, square, pentagon, hexagon and circle. Comparing with these five structures, we obtained the optimal geometric design of the pentamode metamaterials with both excellent pentamodal property and the acoustic band gaps with large bandwidth. And the results can provide practically guidance for large scale experiments.

2. Method and model

The bulk modulus B and shear modulus G of the given structures with diamond lattice can be derived from generalized Hook's law and the elastic wave equations as follows:

$$C_{44} = \rho \left(v_{110}^{T,z} \right)^2 \tag{2.1}$$

$$C_{12} = \rho \left(v_{110}^{L,xy} \right)^2 - C_{44} - \rho \left(v_{110}^{T,xy} \right)^2 \tag{2.2}$$

$$C_{11} = 2\rho \left(v_{110}^{T,xy} \right)^2 + C_{12} \tag{2.3}$$

$$G = C_{44} \tag{2.4}$$

$$\mathrm{B} = (C_{11} + 2C_{12})/3 \tag{2.5}$$

There C_{11}, C_{12} and C_{44} are the three independent components of the fourth-order elastic coefficient tensor for cubic system. v represents the phase velocity of longitudinal and transverse waves. ρ is the mass density of metamaterial, and it is assumed as a scalar constant, which is simply given by $\rho = f\rho_0$, where f is the volume filling fraction of the constituent material.

To obtain the propagation features of elastic wave, the phonon band structure will be computed by the finite-element method (FEM) with the Bloch boundary conditions applied. For high symmetry direction [110] (ΓK), the phase velocities of the transverse and longitudinal waves can be achieved from the slopes of the lowest three acoustic branches that emerge from Γ point of the band structure. Then B and G can be further calculated according to Eqs. (2.1)–(2.5).

The classical pentamode structure is the double-cone arrays with diamond lattice. The primary unit is the double-cone and its cross-section shape is a circle. Inspired by it, we extended the cross-section shape from circle to regular triangle, square, pentagon, hexagon, as illustrated in Fig. 1. To allow for a direct comparison, the regular triangle, square, pentagon and hexagon have the identical diameters of circumcircles with that of the circle for double-cone case. The diameters of circumcircles in thick and thin end are denoted as D and d, respectively. Then the primary units periodically arrange into diamond lattice with lattice constant $a = 37.3$ μm, and the length $H = \sqrt{3}a/4$. The constituent material of units is the polymer with mass density $\rho_0 = 1190$ kg/m^3, Young's modulus $E = 3000$ MPa and Poisson's ratio $\nu = 0.4$.

3. Results

The phonon band structures of pentamode metamaterials with different cross-section shapes have been firstly calculated via FEM. By analyzing and comparing phonon dispersion curves, the effects of geometric feature of primary unit on the pentamodal property and acoustic band gaps of pentamode metamaterials have been studied systematically.

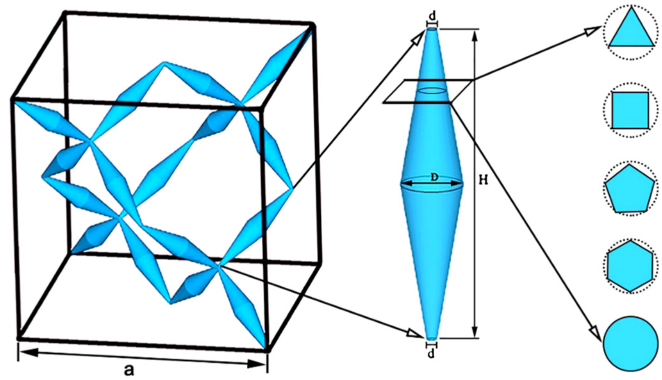

Fig. 1. Illustration of the pentamode structure formed by the periodical arrangement of primary units into diamond lattice. The cross-section shape of primary unit includes regular triangle, square, pentagon, hexagon and circle.

3.1. Pentamodal property

The phonon band structures of these five cases ($D = 2$ μm, $d = 0.25$ μm) have been calculated and the triangle case are shown in Fig. 2(a). The horizontal axis displays the reciprocal lattice vector and the high-symmetry points are highlighted to show the k-path along the border of the first Brillouin zone. The left-hand side vertical axis shows the normalized frequency a/λ, and λ is the wavelength of sound in air with the standard sound velocity of 343 m/s. Results show that these five cases have similar band structures, and the dispersion curves gradually move toward low-frequency direction with the cross-section shapes varying from circle to triangle. The single-mode region highlighted with the light-grey can be found in all these five structures, which means the decouple of the compressive and shear waves. Then the values of B/G are shown in Fig. 2(b) as 2133 for triangle, 1477 for square, 1319 for pentagon, 1182 for hexagon and 1013 for circle, respectively. It is noteworthy that all the B/G have reached to 10^3, which is much larger than that of natural materials. Thus all these five structures can perform well pentamodal property for simultaneously satisfying the two necessary factors of pentamode, the higher B/G and the decouple of compression and shear waves.

Then calculations about the effect of geometrical parameters (D and d) on B/G are carried out. One situation is for fixing D as 2 μm and adjusting d from 0.1 μm to 2 μm, as depicted in Fig. 3(a). The other is for fixing d as 0.25 μm and adjusting D from 0.25 μm to 5 μm, as shown in Fig. 3(b).

The results in Fig. 3(a) show that B/G of these five cases performs similar downtrend with the increasing d. This mainly stems from the higher growth rate of G than that of B with the increase of d. The nonlinear fitting data provide the quantitative relations as $B/G = 135d^{-2}$ for triangle, $B/G = 84d^{-2}$ for square, $B/G = 88d^{-2}$ for pentagon, $B/G = 75d^{-2}$ for hexagon and $B/G = 62d^{-2}$ for circle case. Thus under the same parameter conditions, the triangle case displays the highest B/G and its maximum reaches to 10^4 as $d = 0.1$ μm, which is more than twice as big as that of the circle case.

Fig. 3(b) demonstrates the changing laws of B/G with D for different cross-section shapes. Firstly for $D = d = 0.25$ μm, the values of B/G are 953 for triangle, 954 for square, 952 for pentagon, 948 for hexagon and 961 for circle case, respectively. The fact indicates that these five structures possess almost same B/G and thus the data points at $D = 0.25$ μm nearly overlapped each other. This mainly results from the negligible difference of cross-section shapes for smaller D. Then B/G basically keeps around the respec-