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# Expansion dynamics of Fermi atoms in optical lattice

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A B S T R A C T

We study the non-equilibrium quantum dynamics of attractive Fermi gases in one- and two-dimensional optical lattice. We use the dynamic Bogoliubov–de Gennes (DBdG) method and time-evolving block decimation (TEBD) to investigate the expansion dynamics, which can be implemented by suddenly removing the harmonic trap. The evolutions of density and superfluid order parameters have been calculated. We find that for the noninteracting case, the expansion rate is linear with hopping amplitude, which is a ballistic expansion result. And the interaction damps the expansion rate exponentially both in one and two dimensions and makes it deviate from the ballistic expansion.



## 1. Introduction

Recently, the progress of ultracold atoms in optical lattice have made it possible not only to simulate equilibrium many-body system [1–7], but also to study non-equilibrium quantum system dynamics of many-body systems [8–12]. Differently from condensed matter physics, the extreme low dissipation rate in the ultracold atomic systems guarantees the conservation of the system energy in a relatively long time, thus not only the ground state but also the high energy excited state may contribute to the non-equilibrium dynamics of the many-body systems, which leads to interesting novel phenomena that have no counterpart in condensed matter physics [13–15]. Furthermore, the interaction in ultracold atoms is not only important for the properties of the ground state, but also plays a crucial role in determining the many-body quantum dynamic behavior and has attracted considerable attention recently.

In this paper, we study the dynamic evolution of the attractive interacting Fermi atoms [16–18] in one-dimensional (1D) and two-dimensional (2D) optical lattice after removing the harmonic trap, which is closely related the recent experiments in cold atoms [19–21]. We adopt the dynamic Bogoliubov–de Gennes (DBdG) method and the time-evolving block decimation (TEBD) method to study the non-equilibrium systems. We describe the effect of the interaction on the many-body dynamics. DBdG has been widely used to study the dynamics of the superconductor order parameter after a sudden quench of the interaction strength [22–28]. In this paper, for 2D optical lattice, we adopt DBdG to study the dynamics for the expansion process. Our result shows ballistic transport behavior for the noninteracting gas. And the weak attractive interaction would damp the velocity of the expansion and make it deviate from the ballistic expansion. For the 1D case, TEBD method is adopted to investigate the expansion dynamics. We get a similar result as 2D. And in both cases, the highest expansion rate occurs in the non-interacting limit, where the cloud expands ballistically.

The rest of this paper is organized as follows. In the first part of Section 2, we introduce the calculation method DBdG and briefly discuss its relation with other methods dealing with dynamic processes. In the second part of Section 2, the non-equilibrium processes of expansion in 2D optical lattice are investigated by DBdG method. In Section 3, we adopt TEBD to investigate the expansion dynamics for 1D gas in optical lattice. Finally, we draw some conclusion and discuss our results and recent experimental results.

## 2. 2D case

### 2.1. Model and DBdG method

We consider two components of Fermi gases trapped in 2D deep optical lattice. It is appropriate to describe this system by a single-band Hamiltonian. The total Hamiltonian can be described as follows:

$$H(t) = -J\sum_{\sigma\langle\mathbf{i},\mathbf{i}'\rangle}(c^\dagger_{\mathbf{i}\sigma}c_{\mathbf{i}'\sigma}+h.c.)+g\sum_{\mathbf{i}}c^\dagger_{\mathbf{i}\uparrow}c^\dagger_{\mathbf{i}\downarrow}c_{\mathbf{i}\downarrow}c_{\mathbf{i}\uparrow} + \sum_{\mathbf{i}\sigma}(V_{\mathrm{trap}}(\mathbf{i},t)-\mu_\sigma)c^\dagger_{\mathbf{i}\sigma}c_{\mathbf{i}\sigma} \quad (1)$$

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$c_{\mathbf{i}\sigma}$ ($c^\dagger_{\mathbf{i}\sigma}$) is the creation (annihilation) operator of Fermi atoms with spin $\sigma$ at site $\mathbf{i}$ in 2D optical lattice. Here, we do not consider the situation of imbalanced fermions, so the chemical potential $\mu_\uparrow = \mu_\downarrow = \mu$. $J$ is the amplitude of the hopping, and $g$ is the on-site interaction strength. $V_{\text{trap}}(\mathbf{i}, t)$ describes the time-dependent trap potential of site $\mathbf{i}$. Since we are interested in the weak attractive interacting system, it is proper to use mean field approximation and the origin Hamiltonian changes to:

$$H_{\text{BCS}}(t) = -J\sum_{\sigma\langle\mathbf{i},\mathbf{i}'\rangle}(c^\dagger_{\mathbf{i}\sigma}c_{\mathbf{i}'\sigma} + h.c.) + \sum_{\mathbf{i}}(\Delta_{\mathbf{i}}(t)c^\dagger_{\mathbf{i}\uparrow}c^\dagger_{\mathbf{i}\downarrow} + \Delta^\dagger_{\mathbf{i}}(t)c_{\mathbf{i}\downarrow}c_{\mathbf{i}\uparrow}) + \sum_{\mathbf{i}\sigma}(V_{\text{trap}}(\mathbf{i}, t) - \mu_\sigma)c^\dagger_{\mathbf{i}\sigma}c_{\mathbf{i}\sigma} \quad (2)$$

Here, $\Delta_{\mathbf{i}}(t) = -g\langle c_{\mathbf{i}\downarrow}c_{\mathbf{i}\uparrow}\rangle$, is the pairing parameter for BCS state at time $t$. We describe the dynamic process in the Heisenberg picture, which means the state is not changed while the operator is evolving under the mean field Hamiltonian. The total procedure of our calculation can be summarized as the following: for $t = 0$, the waves function of ground state and all the correlation functions such as $\langle\psi_0|c^\dagger_{\mathbf{i}\sigma}c_{\mathbf{i}'\sigma'}|\psi_0\rangle$ can be derived by Real Space Bogoliubov–de Gennes (RBdG) method [29–31]; when $t > 0$, since the superfluid order parameter $\Delta_{\mathbf{i}}(t)$ in the Hamiltonian is time dependent, we should replace it by the new result step by step during the evolution process. And the chemical potential $\mu$ is updated by the fixed particle number condition. Meanwhile, the single-particle correlation function for the new time point can be evolved by the Hamiltonian of the previous step.

There is another approach to the dynamics of BCS state, the time-dependent Ginzburg–Landau equation [32], which describes the evolution of BCS order. However, it is suitable close to the transition temperature. Since we are interested in non-equilibrium dynamics with quench trap potential at zero temperature, we adopt the method introduced above to deal with the problem. It should be pointed out that, the starting point of this method is the same with the former works in the k-space [22–28], since the mean field approximation is employed in all the calculation. However, we adopt the calculation in the real space due to our interest here.

### 2.2. Dynamics of expansion process

We perform our calculation based on the recent experimental setup. The expansion process is after the sudden movement of the harmonic trap at $t = 0^+$ [19]:

$$V_{\text{trap}}(\mathbf{i}, t) = V_0(\mathbf{x_i} - \mathbf{x}_0)^2\Theta(-t) \quad (3)$$

Here, $\mathbf{x}_0$ is the coordinate of the trap center. $V_0$ is the strength of the external harmonic trap. At $t = 0$, the system is at the ground state (GS) in the trap. For large trap depth $V_0 = 1.5$, atoms in the trap center is in band insulate state, initially. When the trap is removed at $t = 0^+$, the atoms confined initially are allowed to expand freely in the optical lattice. The expansion processes with and without interaction are both studied in our calculation. The density distribution at different time during the expansion process is shown in Fig. 1. For the situation with (without) attractive interaction, as shown in the right (left) column in Fig. 1, we can see the density profile has been affected by the shape of lattice during the free expansion no matter whether the interaction exists.

Furthermore, we show the distribution of BCS pairing order $\Delta(\mathbf{x}, t)$ in the dynamic process in Fig. 2. As we have mentioned above, the majority of the atoms are in the band insulating state, initially. Thus only the fermions in the edge can exhibit coherent pairing. When the trap is removed at $t > 0$, free expansion destroys band insulate state and enhances the coherence. It leads to two effects on superfluid order. On one hand, $\Delta(\mathbf{x})$ expands towards the trap center gradually as a result of melting. On the other hand, with the diffusion of particle density, the peak value of pairing parameter descends.

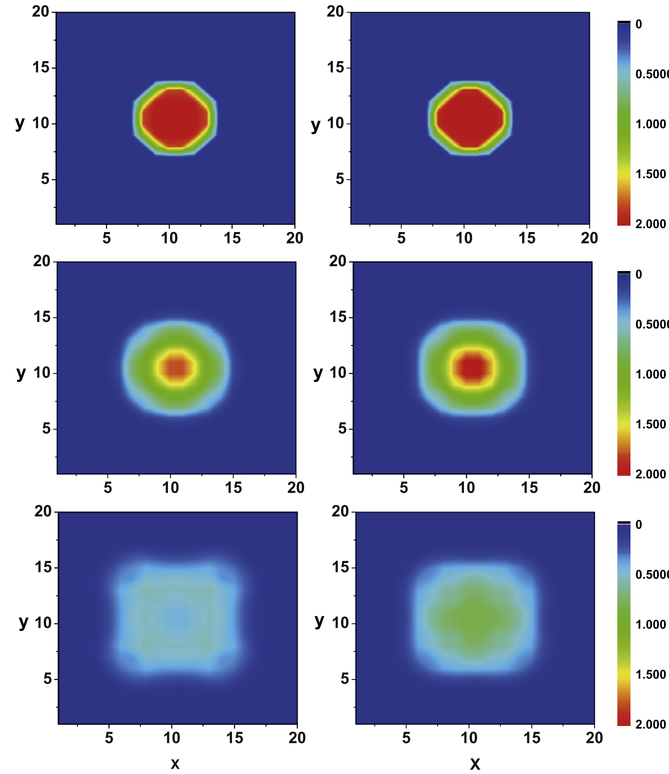


**Fig. 1.** Distribution of density at different time during expansion for noninteracting fermions (left column) and attractive fermions (right column, interaction parameter $g = -2$) with initial trap depth ration $V_0 = 1.5$ and fixed particle number $N = 26$. The rows display the density for $t = 0$, $t = 1$, and $t = 2$, from top to the bottom respectively. And the time unit is $\hbar/J$.

In order to describe the expansion process, we adopt the concepts of cloud radius and mean expansion rate, which have been defined in [19]:

$$R(t) = \sqrt{R_0^2 + v_{\text{exp}}^2 t^2} \quad (4)$$

$R_0$ and $R(t)$ are radius of the cloud at initial and at real-time, respectively, which satisfy $R_0^2 = \frac{1}{N}\sum_i n_i(0)(i - i_0)^2$ and $R_t^2 = \frac{1}{N}\sum_i n_i(t)(i - i_0)^2$. And the deconvolved cloud size which gets rid of the initial cloud size satisfies $\underline{R_t} = \sqrt{R_t^2 - R_0^2}$. Here, $N$ is the total particle number, and $i_0$ is the coordinate of the trap center. The lattice parameter $a$ is set as 1 in the above equations. Total cloud size and deconvolved size are obtained from the above equations, shown in Fig. 3. For non-interacting expansion, the process is ballistic expansion. We find the expansion rate is linear with the hopping amplitude $J$, which is accordant with the result of experiment measurement, as shown in the insert in Fig. 3. It is simple to explain this result. Due to the band insulate state, the atom is localized completely while all the moment k-states are populated. A mean squared velocity $v_{\text{exp}}^2 = \langle\mathbf{v_q}^2\rangle$. $\mathbf{v_q} = \frac{1}{\hbar}\frac{\partial E}{\partial\mathbf{q}}$. After simple calculation, we can get the result $v_{\text{exp}} = \frac{2J}{\hbar}$. Furthermore, the deconvolved cloud size shows a linear relation with the expansion time for the ballistic expansion, while the total cloud size shows a parabolic curve vs expansion time. If attractive interaction is introduced, the deconvolved cloud size vs time $t$ deviates from the linear relation. And the cloud size of the interacting case is smaller