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Mott-superfluid transition of q-deformed bosons

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ABSTRACT

The effect of q-deformation of the bosonic algebra on the Mott-superfluid transition for interacting lattice bosons described by the Bose–Hubbard model is studied using mean-filed theory. It has been shown that the Mott state proliferates and the initial periodicity of the Mott lobes as a function of the chemical potential disappears as the q-deformation increases. The ground state phase diagram as a function of the q-parameter exhibits superfluid order, which intervenes in narrow regions between Mott lobes, demonstrating the new concept of statistically induced quantum phase transition.

1. Introduction

The phenomenon of spontaneously broken symmetry is one of the most fascinating and fundamental problem relevant for solids, ultra-cold atoms, and relativistic physics. The experimental discovery of Bose–Einstein condensation (BEC) has opened up the study of quantum phenomena in the context phase transition in the in a qualitatively new regime [1,2]. The research on ultra-cold matter was further accelerated by successful experiments on BECs in optical lattices [3], which are created by periodic Stark shift potentials resulting from the interference of two or more laser beams. Interestingly, due to its universality, BEC in optical lattices may be employed for simulation of many-body quantum physics which is common in condensed matter physics. Advances in the technology of engineering many-body systems with cold atoms trapped in optical lattices allow for building quantum simulators, i.e., systems with model quantum Hamiltonians, where types of interactions can be customized and their strengths tuned. In particular, cold atoms in optical lattices realize Hubbard dynamics for both bosonic and fermionic particles, where the single particle and interaction terms can be engineered by external fields. Especially, there has been a lot of interest in the possibility of building quantum simulators for lattice gauge theory (LGT) using optical lattices [4–6]. The purpose is to experimentally engineer many-body systems with cold atoms that approximately evolve according to some given quantum LGT Hamiltonian. In the context of condensed matter, a proof that quantum simulating is possible has been given in the case of the Bose–Hubbard (BH) model [7], where an impressive level of quantitative agreement has been reached between theoretical calculations and their experimental optical lattice implementations.

From another perspective superfluid–insulator transition rests on spin-statistics theorem which represents one of the fundamental principles of physics establishing a connection between quantum mechanics of many-body systems. In the recent years there has been increasing emphasis in quantum statistics different from the standard bosons and fermions [8]. One interesting realization of this approach is to study of statistical by employing the q-deformed algebra of creation and annihilation operators, usually called q-bosons [9,10]. The theory of q-deformed bosons is related to the general theory of quantum groups and originated from the study on exactly solvable statistical systems, which led to the q-deformed algebra of creation and annihilation operators [11,12]. In this context the deformation parameter q can be considered as an effective quantity which encapsulates most features of non-trivial dynamics of the system under study.

In a recent studies of the bosonic optical lattice system an experimental setup was proposed to create anyons in one-dimensional lattices with fully tunable exchange statistics [13]. Here, anyons were created by bosons with occupation-dependent hopping amplitudes, which can be realized by assisted Raman tunneling. The system is described as a variant of the Bose–Hubbard model where the bosonic hopping amplitudes are state-dependent with the conditional-hopping phase factor that breaks reflection parity in the system which is an important ingredient of fractional statistics. This opens possibility of smoothly transmuting bosons via anyons into fermions since the particles operators at lattice sites k, j obey the deformed commutation relation

$$\hat{a}_k\hat{a}_j^\dagger - q_{kj}\hat{a}_j^\dagger\hat{a}_k = \delta_{kj} \tag{1}$$

with the deformation parameter $q_{kj} = e^{i\theta\,\mathrm{sgn}(k-j)}$. The sign function is such that $\mathrm{sgn}(k-j) = 0$ for $j = k$. The statistical angle θ can be

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controlled by modifying the relative phase of external driving fields via assisted Raman tunneling which can selectively address hopping processes connecting different occupational states and induce a relative phase, realizing a fully tunable deformation parameter q [13]. The case $q = 1$ corresponds to ordinary bosons, however for $\theta = \pi$ they behave as pseudo-fermions: while being bosons on-site, they are fermions off-site. Thus by changing the deformation parameter q one can demonstrate the new concept of *statistically induced* quantum phase transition.

The aim of this paper is to provide further step towards a theoretical understanding of the properties of interacting bosons under the q deformation of the fundamental Bose–Einstein algebra. While the statistical properties of q-deformed non-interacting bosonic particles have been examined to some extent [14–17] the physics of interacting deformed counterparts remain elusive. To this end, we investigate the finite temperature phase diagram of strongly interacting lattice q-bosons described by Bose–Hubbard model, with the aim to study the impact of the q deformation on the ground state and finite temperature properties of the system with special emphasis on the Mott-superfluid transition.

In order to handle system with strong local interactions the resolvent technique based on the contour integral representation of the partition function has been devised [18]. Subsequently, we derive the Landau-type expansion for the free energy in terms of the superfluid order parameter and find the phase diagrams depicting the relationships between various physical quantities of interest.

2. The model

The Bose–Hubbard model [7], which incorporates strong correlations of the many-body system and successfully captures the Mott transition, is utilized as a generic model for the many-body system in an optical lattice. In the BH model, the low-lying excitations are described by the motion of a fundamental boson from a site to a neighboring site. To move a fundamental boson from a site to a neighboring lattice site costs energy U because of the repulsive Coulomb force between the fundamental bosons. In second quantized form the BH Hamiltonian is given by

$$\mathcal{H} = \sum_i (\hat{V}_i - \mu \hat{n}_i) - J \sum_{\{ij\}} (\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i)$$
$$\hat{V}_i = \frac{U}{2} \hat{n}_i (\hat{n}_i - 1) \tag{2}$$

with experimentally adjustable ratios between the hopping amplitude J between pairs of lattice sites labeled by $\{ij\}$ and the on-site interaction U. The chemical potential μ is added to fix the number of particles in the grand canonical ensemble. Here, $\hat{a}_i$ is the bosonic field operator at i-th site of the lattice, $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$ is the particle number operator for bosons. At integer filling factor, and zero temperature it describes a second-order quantum phase transition from superfluid (SF) to the Mott insulator (MI) phase as the interaction strength U is increased. In the Mott insulator phase, bosons are localized on the lattice sites due to the repulsive interaction and they do not form a coherent state. In the coherent superfluid phase, bosons are delocalized and an long-range order exists. Furthermore, at finite temperature T, in addition to quantum fluctuations one has to consider the thermal fluctuations with an energy scale of kT, with k being the Boltzmann constant, which can influence the properties of the many-body system.

3. *q*-deformation of Bose operators

In what follows, we shall exploit the q-Bose gas picture based on a concrete version of deformed bosons with the deformation parameter q. We employ the formulation which is symmetric under $q \longleftrightarrow q^{-1}$ [9,10]. In terms of the q parameter an arbitrary real value can be considered. In this symmetric formulation one can further restrict it to $0 < q < 1$ (or equivalently to $1 < q < \infty$. A q-boson algebra is a set of elements called q-boson operators: $\hat{a}$ (annihilation), $\hat{a}^\dagger$ (creation) and $\hat{\mathcal{N}}$ (number), which satisfy the following commutation relations

$$[\hat{\mathcal{N}}, \hat{a}^\dagger]_q = \hat{a}^\dagger, \quad [\hat{\mathcal{N}}, \hat{a}]_q = -a$$
$$\hat{a}\hat{a}^\dagger - q\hat{a}^\dagger\hat{a} \equiv [\hat{a}, \hat{a}^\dagger]_q = q^{\hat{\mathcal{N}}}. \tag{3}$$

The first and second commutation relations are that for bosons, however, the third depends on the parameter q and only when $q = 1$ we recover the ordinary bosonic commutation rules. The action of the deformed bosonic operators on the basis is given by the rules

$$\hat{a}^\dagger |n\rangle = \sqrt{[n+1]_q}\, |n+1\rangle$$
$$\hat{a}|n\rangle = \sqrt{[n]_q}\, |n-1\rangle$$
$$\hat{\mathcal{N}}|n\rangle = n|n\rangle \tag{4}$$

which are similar to those of ordinary bosons, the only difference being the q-number given by

$$[x]_q = (q^x - q^{-x})/(q - q^{-1}), \tag{5}$$

which appears under the square roots instead of common numbers and satisfies the non-additivity property

$$[n+1]_q = q[n]_q + q^{-n}. \tag{6}$$

The Fock space is constructed by allowing polynomials in the creation operator to act on the vacuum

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{[n]_q!}} |0\rangle \tag{7}$$

where the q-basic factorial is defined as

$$[n]_q! = [n]_q [n-1]_q [n-2]_q \ldots [1]_q. \tag{8}$$

Finally, note that the number operator $\hat{\mathcal{N}}$ differs from $\hat{n}$ operator, but the relation between them can be expressed as the nonlinear functional form $\hat{n} = [\hat{\mathcal{N}}]_q$ or equivalently

$$\hat{\mathcal{N}} = \frac{1}{\ln q} \ln \left[\frac{\hat{n}(q^2-1) + \sqrt{4q^2 + \hat{n}^2(q^2-1)^2}}{2q} \right] \tag{9}$$

Obviously, in the non-deformation limit $q \to 1$, the q-basic number $[x]_q$ reduces to the ordinary number x and the relation $\hat{n} \equiv \hat{\mathcal{N}}$ is restored.

4. Mean-field theory

In this section, using the mean-field method we calculate the thermodynamic energy for BH Hamiltonian by applying perturbation theory. For the problem under study, the superfluid state spontaneously breaks the U(1) symmetry of the model in Eq. (2) as a result of the non-vanishing order parameter

$$\Phi = \langle \hat{a}_i \rangle, \tag{10}$$

where $\langle \ldots \rangle$ denotes statistical average. Due to the gauge U(1) symmetry the parameter Φ can be considered as a real number. The mean-field method make use of the representation

$$\hat{a}_i = \langle \hat{a}_i \rangle + \hat{\delta}_i = \Phi + \hat{\delta}_i, \tag{11}$$