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Mean-field solution of the Blume–Capel model under a random crystal field

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ABSTRACT

In this work we investigate the Blume–Capel model with infinite-range ferromagnetic interactions and under the influence of a quenched disorder – a random crystal field. For a suitable choice of the random crystal field the model displays a wealth of multicritical behavior, continuous and first-order transition lines, as well as re-entrant behavior. The resulting phase diagrams show a variety of topologies as a function of the disorder parameter p. A comparison with recent results on the Blume–Capel model in random crystal field is discussed.

1. Introduction

In recent years there has been increasing interest in the multicritical behavior of disordered systems. Special attention has been given to models with the inclusion of random fields, in the case of disordered magnetic systems, both for theoretical interest and also for its correspondence with the experimental results [1]. Among those models, the Blume–Capel model [2,3] and some of it extensions have received a lot of attention. The Blume–Capel is itself an extension of the classical Ising model for spin-1 which takes into account the effect of a local crystal field anisotropy. Its phase diagram displays a continuous transition line which meets a first-order transition line at a tricritical point [4]. From the theoretical point of view a particularly interesting question is how such phase diagrams are changed under the effect of quenched randomness [5–8]. Because of that, Kaufman and Kanner [9] studied the Blume–Capel model under a random magnetic field and obtained a rich variety of phase diagrams. The effect of random crystal field has been considered by several authors [10–26]. Besides the approach adopted, in some of these works the choice of the random crystal field distribution is also different. However, in all cases the phase diagrams display a rich behavior with the presence of critical and coexistence lines, as well as many multicritical points and re-entrant phenomena. In their recent work, Salmon and Tapia studied an infinite-range Blume–Capel under a quenched disorder crystal field following a superposition of two Gaussian distributions and classified the phase diagrams according to their topology (scenario) [21]. Recently, the effect of a special discrete random crystal field distribution was investigated by the pair approximation approach [26]. The same type of random crystal field distribution had already been investigated by the real-space renormalization-group approach, as well as by the mean-field approximation [18,20]. However, as far as the results can be compared they lead to qualitatively different phase diagrams for low temperature. For instance, while the pair approximation predicts first-order transitions between the paramagnetic and ferromagnetic at zero temperature as in Fig. 2 of [26], the conclusion of the single-site mean-field approximation is that the ground state is always ordered according to Eq. (3) of [20]. Thus, we decided to investigate this point further by considering an exactly soluble version of the Blume–Capel model under the random crystal field distribution considered by [18,20,26]. Besides, we are also interested in investigation the possible topologies for phase diagrams predicted by this sort of mean-field treatment along the lines of the continuous distribution started by [21]. Finally, since the re-entrant phenomena in random spin-1 models have attracted some recent interest (see, for instance, [27] and references therein), our results may give some hint as to what expect in more disordered system as in the case of the Blume–Capel spin-glass under a random crystal field [28].

This work is organized as follows. In Section 2 we introduce the Blume–Capel model under the presence of a random crystal field and obtain the basic equations. In Section 3 we present the

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obtained phase diagrams. Finally, we present our conclusions in Section 4.

2. The model

Let us consider the infinite-range Blume–Capel model described by the following Hamiltonian:

$$\mathcal{H} = -\frac{J}{2N}\sum_{(i,j)}^{N} S_i S_j + \sum_{i=1}^{N} \Delta_i S_i^2 \,, \tag{1}$$

where N is the number of spins and $S_i = -1, 0, +1$, for all sites $i = 1, \cdots, N$. The first sum runs over all pairs of spins (i, j). The ferromagnetic coupling takes the form J/N to account for the free energy extensivity. The random crystal fields Δ_i are quenched variables, independent and identically distributed according to the following probability distribution:

$$P(\Delta_i; D, p) = p\delta(\Delta_i - D) + (1-p)\delta(\Delta_i + D) \,. \tag{2}$$

As far as we know, the above probability distribution was introduced by Branco and Boechat [18] and Branco [20] and has been recently considered by Lara [26]. The transformation $\Delta_i' = (\Delta_i + D)/2$ leads to the probability distribution mostly used in the study of the Blume–Capel in discrete random crystal field as, for instance, in [10,13], but also produces a slight change in the Hamiltonian. The general properties of the phase diagram should not depend on the particular form of the discrete random crystal field distribution. Thus, besides our interest in making comparison with known results [18,20,26], another reason for our choice of the probability distribution is the symmetry inherent in Eq. (2) which can be expressed by:

$$P(\Delta_i; D, p) = P(\Delta_i; -D, 1-p). \tag{3}$$

Therefore, in order to determine phase diagrams for fixed values of p it is sufficient to consider the domain defined by $D \geq 0$ and $1/2 \leq p \leq 1$. By symmetry considerations we can obtain the corresponding phase diagrams for $0 \leq p \leq 1/2$ by a mapping from $1/2 \leq p \leq 1$.

Using the replica method (see Appendix A for details), we obtain the free-energy density, in units of J:

$$\begin{aligned} f(t, d, p; m) = \frac{1}{2}m^2 &- pt\ln\left[1 + 2\exp(-d/t)\cosh(m/t)\right] \\ &- (1-p)\,t\ln\left[1 + 2\exp(d/t)\cosh(m/t)\right], \end{aligned} \tag{4}$$

where $t = k_B T/J$, $d = D/J$, and m is the magnetization. The equation of state can be obtained by taking the minimum of the above free-energy functional with respect to m, which leads to

$$m = \frac{2p\sinh(m/t)}{\exp(d/t) + 2\cosh(m/t)} + \frac{2(1-p)\sinh(m/t)}{\exp(-d/t) + 2\cosh(m/t)}. \tag{5}$$

The thermodynamic properties of the model are completely determined by Eqs. (4) and (5) which, in turn, reveals clearly the symmetry expressed by Eq. (3). For given values of p, t and d the physical solution corresponds to the global minima of the free-energy density. Thus, for a given value of p we can determine the d–t phase diagram. Eq. (5) always has a trivial solution corresponding to the paramagnetic phase **P**, with $m = 0$. The corresponding paramagnetic free-energy density is given by

$$\begin{aligned} f_P(t, d, p) = &-pt\ln\left[1 + 2\exp(-d/t)\right] \\ &- (1-p)\,t\ln\left[1 + 2\exp(d/t)\right]. \end{aligned} \tag{6}$$

Besides the paramagnetic solution, Eq. (5) may present distinct non-trivial solutions, corresponding to different ferromagnetic phases.

Let us consider the ground state. For $d > 0$, the free-energy density f_P for the paramagnetic solution becomes

$$f_0 \equiv f_P(t = 0, d, p) = -(1-p)\,d. \tag{7}$$

Apart from the paramagnetic phase, we find two ferromagnetic solutions. The first type (**F1**) is characterized by $m_1 = 1$, with the free energy density given by:

$$f_1 \equiv f(t = 0, d, p; m = 1) = -\frac{1}{2} + (2p-1)\,d, \quad \text{for } d < 1. \tag{8}$$

The second type of ferromagnetic solution (**F2**) is given by $m_2 = 1 - p$, with the free energy density given by:

$$f_2 \equiv f(t = 0, d, p; m = 1-p) = -\frac{1}{2}(1-p)^2 - (1-p)d, \quad \text{for } d \geq 1-p. \tag{9}$$

From Eqs. (7) and (9), we note that $f_2 \leq f_0$ wherever the **F2** phase exists. Moreover, from the analysis of Eqs. (7)–(9) we find that the ground state consists of the **F1** phase for $d < d_0$, while for $d > d_0$ it corresponds to the **F2** phase. At zero temperature, $t = 0$, we determine a first-order transition between the **F1** and **F2** phases at d_0 given by

$$d_0 = 1 - \frac{1}{2}p. \tag{10}$$

Therefore, except for p strictly equals to 1 the paramagnetic phase is never realized at zero temperature.

In general the d–t phase diagrams for a given value of p can be determined numerically from Eqs. (4) and (5). However, the stability of the paramagnetic phase can be determined analytically. From this analysis we can find critical frontiers as well as possible tricritical points. For this purpose, let us introduce the following parametrization:

$$a = \exp(d/t). \tag{11}$$

Nearby a continuous transition from ferromagnetic to paramagnetic phase, we consider a small magnetization $m \simeq 0$ and write a Landau-like expansion for the free energy density:

$$f(t, d, p; m) = A_0 + A_2 m^2 + A_4 m^4 + A_6 m^6 + \cdots. \tag{12}$$

The coefficient A_0 corresponds to $f_P(t, d, p)$ given by Eq. (6), while the remaining coefficients are given by:

$$A_2 = \frac{1}{2} - \frac{p}{2t}q - \frac{(1-p)}{2t}r, \tag{13}$$

$$A_4 = -\frac{p}{24t^3}(1-3q)q - \frac{(1-p)}{24t^3}(1-3r)r, \tag{14}$$

$$\begin{aligned} A_6 = &-\frac{p}{720t^5}(1 - 15q + 30q^2)q \\ &- \frac{(1-p)}{720t^5}(1 - 15r + 30r^2)r, \end{aligned} \tag{15}$$

where q and r are given by

$$q = \frac{2}{2+a}, \quad r = \frac{2a}{2a+1}. \tag{16}$$

These new parameters q and r are not independent and can be interpreted as the density of spins $S_i = \pm 1$ in the paramagnetic phase for the pure cases $p = 1$ and $p = 0$, respectively.

A continuous transition line from the ferromagnetic to paramagnetic phase is given by

$$A_2 = 0, \quad \text{while } A_4 > 0.$$