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Single qudit realization of the Deutsch algorithm using superconducting many-level quantum circuits

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ABSTRACT

Design of a large-scale quantum computer has paramount importance for science and technologies. We investigate a scheme for realization of quantum algorithms using *noncomposite* quantum systems, *i.e.*, systems without subsystems. In this framework, n artificially allocated "subsystems" play a role of qubits in n-qubits quantum algorithms. With focus on two-qubit quantum algorithms, we demonstrate a realization of the universal set of gates using a $d = 5$ single qudit state. Manipulation with an ancillary level in the systems allows effective implementation of operators from U(4) group via operators from SU(5) group. Using a possible experimental realization of such systems through anharmonic superconducting many-level quantum circuits, we present a blueprint for a single qudit realization of the Deutsch algorithm, which generalizes previously studied realization based on the virtual spin representation (Kessel et al., 2002 [9]).

1. Introduction

Building of a large-scale quantum computer is one of the most challenging domains of quantum information technologies [1–6]. This new generation of computational devices demonstrates a potential to outperform their classical counterparts greatly [2–6]. Examples include searching an unsorted database [5] as well as integer factorization and discrete logarithm problems [6] to name a few.

From a physical point of view, a quantum computer is an open quantum system with a large number of subsystems, which play a role of information units. These systems can be realized via a variety of physical platforms. Quantum states of a composite system are described by the density operator in the abstract Hilbert space being a product,

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \cdots \otimes \mathcal{H}_Z, \quad (1)$$

of the Hilbert spaces of the physical subsystems. A crucial requirement to such systems as platforms for quantum information processing is scalability with respect to number of qubits [7]. Success in scalability of the systems results in increasing the number of subsystems making the problem of achieving a suitable degree of control more and more challenging.

However, a set of required features for quantum technologies is available not only in composite systems but in noncomposite systems as well [8–14]. Recent experimental study of photonic qutrit states demonstrates fundamentally non-classical behavior of noncomposite quantum systems [10]. The idea behind this result dates back to the Kochen–Specker theorem [15], which provides certain constraints on hidden variable theories, that could be used to explain probability distributions of quantum measurement outcomes.

The Hilbert space of noncomposite systems is arranged in the opposite way to (1), however it is equivalent to that mathematically: it can be represented in form (1), *i.e.*, as a product of the Hilbert spaces of *abstract* subsystems. Investigations of information and entropic characteristics of noncomposite quantum systems [11–14] have confirmed possibilities of their applications in quantum technologies. Furthermore, a potential gain from the use of noncomposite many-level quantum systems has been demonstrated in quantum coin-flipping and bit commitment [16], protocols for quantum key distribution [17–20], quantum information processing [8,9,21–24] and clock synchronization algorithms [25].

Remarkably, these studies are supported by substantial progress in experiments with many-level states of photons [26], trapped

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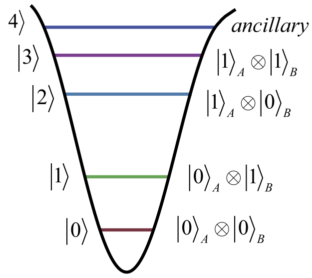

Fig. 1. Mapping of a five-level quantum system on a two-qubit quantum system.

ions [27], NMR setups [24], and superconducting quantum circuits [28–32].

In the present work, we stress on the implementation of quantum algorithms via noncomposite quantum systems with focus on their realization via anharmonic superconducting many-level quantum circuits using addressing to a particular transition. Our consideration is valid for an arbitrary realized many-level quantum system. However, we focus on many-level superconducting circuits due to significant progress in their design [28–32].

These advances allow to create highly tunable artificial atomic systems with possibilities to reproduce interesting quantum effects [33–41] as well as employ them for quantum computing [42–46] and simulation [47].

Recent experiment with a superconducting four-level quantum circuit has explored "hidden" two-qubit dynamics [31]. Therefore, it is interesting to study possibilities of demonstration computational speed-up from single qudit realization of oracle-based algorithms using superconducting many-level circuits.

Here, we consider a qudit state with $d=5$, where four levels are used for storage of two-qubit quantum states and an ancillary fifth level is employed for effective realization of operators from U(4) group via operators from SU(5) group (see Fig. 1). We demonstrate that this trick makes it possible to construct the universal set of two-qubit quantum gates consisting of Hadamard, $\pi/8$ and controlled NOT gates [48].

The main emphasis of our work is on a single qudit realization of one of the first oracle-based quantum algorithm — the Deutsch algorithm [49]. Employment of the ancillary level is a novel feature compared to our previous study [50], where we considered a $d=4$ qudit state and proposed a scheme for Hadamard gates from the universal set only, as well as with previously studied realization of the Deutsch algorithm [9]. The suggested single qudit realization of the Deutsch algorithm differs from previously studied [9], where the operated physical environment allowed to apply arbitrary quantum gates without using of ancillary levels.

Our paper is organized as follows. In Section 2, we consider a correspondence between a qudit state with $d=5$ and a two-qubit quantum system as well as propose scheme for the universal set of quantum gates for two-qubit algorithms using noncomposite quantum systems. Using the universal set of quantum gates, we present a realization for a single qudit realization of the Deutsch algorithm in Section 3. We conclude the paper and summarize results in Section 4.

2. Universal set of gates

The composite representation of noncomposite quantum d-level systems with $d>2$ corresponds to any possible mapping of its Hilbert space on a tensor product of several Hilbert spaces, which correspond to abstract subsystems.

In this paper, we consider the five-dimensional Hilbert space of anharmonic superconducting many-level quantum circuit (see Fig. 1). The correspondence between the stationary energy states and two-qubit logic basis can be presented as follows:

$$|0\rangle \to |0\rangle_A \otimes |0\rangle_B, \quad |1\rangle \to |0\rangle_A \otimes |1\rangle_B,$$
$$|2\rangle \to |1\rangle_A \otimes |0\rangle_B, \quad |3\rangle \to |1\rangle_A \otimes |1\rangle_B. \qquad (2)$$

This mapping resembles the virtual spin representation suggested in Ref. [8]. We assume that the population of the fifth level is negligible and we keep them in consideration only for the implementation of quantum gates.

Due to above-stated assumptions, the state of the system, written in the original basis, can be presented as

$$\rho \equiv \rho_{AB} = \begin{bmatrix} \rho_{00} & \rho_{01} & \rho_{02} & \rho_{03} & 0 \\ \rho_{01}^* & \rho_{11} & \rho_{12} & \rho_{13} & 0 \\ \rho_{02}^* & \rho_{12}^* & \rho_{22} & \rho_{23} & 0 \\ \rho_{03}^* & \rho_{13}^* & \rho_{23}^* & \rho_{33} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (3)$$

while the states of allocated "subsystems" A and B turn to have a form

$$\rho_A = \begin{bmatrix} \rho_{00}+\rho_{11} & \rho_{02}+\rho_{13} \\ \rho_{02}^*+\rho_{13}^* & \rho_{22}+\rho_{33} \end{bmatrix},$$
$$\rho_B = \begin{bmatrix} \rho_{00}+\rho_{22} & \rho_{01}+\rho_{23} \\ \rho_{01}^*+\rho_{23}^* & \rho_{11}+\rho_{33} \end{bmatrix}, \qquad (4)$$

where the matrices are written in their corresponding computational bases.

We assume that our toolbox the system manipulation consists of applying θ-pulses on the transition between arbitrary pair of energy levels. In general, it can be done via coupling of a superconducting many-level quantum circuit to an external resonant field [28–32].

The corresponding elementary procedure turns to be rotation around X-axis of the "Bloch sphere" of the particular two-dimensional Hilbert subspace:

$$R_X^{(jk)}(\theta) = \begin{bmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}^{(jk)} \oplus \mathbb{I}_3^{(\overline{jk})}, \qquad (5)$$

where the matrix superscript $j,k \in \{0,1,2,3,4\}$ indicates that it is written in the basis $\{|j\rangle, |k\rangle\}$, $\oplus$ stands for the direct sum, $\mathbb{I}_n$ stands for the identity operator in n-dimensional Hilbert space and superscript $(\overline{jk})$ indicates that the identity operator acts in the orthogonal complement $(\mathrm{Span}\{|j\rangle, |k\rangle\})^\perp$, then $R_X^{(jk)}(\theta)$ acts in the whole original five-dimensional Hilbert space.

The appropriate sequence of rotations around X-axis results in the effective rotation around Y-axis:

$$R_Y^{(jk)}(\theta) = \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}^{(jk)} \oplus \mathbb{I}_3^{(\overline{jk})}$$
$$= R_X^{(jl)}(\pi)\, R_X^{(kl)}(\theta)\, R_X^{(jl)}(3\pi), \qquad (6)$$

where the l-th level is one from $(\overline{jk})$. We note that (5) and (6) correspond to SU(5) group of unitary operations with the unit determinant.

It is well-known [48] that for the case of two-qubit systems the universal set of gates consists of one-qubit Hadamard and $\pi/8$-gates,

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 0 \\ 0 & \exp(i\pi/4) \end{bmatrix}, \qquad (7)$$

together with two-qubit controlled NOT gates,

$$U_{\mathrm{CNOT}}^{(A\to B)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad U_{\mathrm{CNOT}}^{(B\to A)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \qquad (8)$$