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On a study of optically coupled memristive Chua circuits—rhythmogenesis and amplitude death

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ABSTRACT

Properties of memristive inductorless Chua circuits are studied when they are coupled optically to characterize the oscillation quenching phenomenon of amplitude death (AD) and oscillation generation procedure of rhythmogenesis. The behaviors of these systems, when studied under coupled condition, show some new features which are not seen previously. This phenomenon is really a novel one as it is the generation of oscillation due to the interaction of two such systems each at their respective steady states. The other event is amplitude death (AD) observed by increase in the coupling strength. The numerical simulation is supported with the data obtained via analogue circuit implementation of the system. Two circuits coupled through a LED (light emitting diode) and LDR (photo resistor) pair show transition to chaotic state under parameter variation. The experimental data was collected with the help of digital to analog converter system. Our data indicates that there exist two different routes to chaos—either through period doubling or without it.

1. Introduction

A few years back in a seminal paper S. Smale [1] considered a very interesting situation when two nonlinear systems at their steady states being coupled lead to an oscillatory situation. This has got an extremely important relevance in biological situations. Such biological systems were primarily studied by Alan Turing [1]. On the other hand, many non-linear systems show the oscillation quenching phenomenon of amplitude death (AD). These two phenomena can be considered being reverse of each other. Here our motivation is to analyze both of these in a single situation of two optically coupled Chua circuits and show that various interesting events unfolds either through numerical simulations or by experiments with the help of analog circuits.

In recent years there has been a surge of activities in the study of analogue circuits for the simulation of nonlinear dynamical systems through experiment. One such famous system which was actually suggested from the circuit itself is the Chua circuit [2]. This circuit consists of capacitors, inductors, and a nonlinear Chua diode. In this connection it may be mentioned that in 1971 [3], it was L.O. Chua who first proposed that there should be a fourth circuit element other than the three known ones, resistance (R), inductance (L) and capacitance (C). This he called memristor (M) to indicate that it is some kind of resistor with a memory [4]. One may call that though it was proposed long ago but only in the year 2008 [5], Hewlett–Packard announced that a fabrication has become a possibility but still not commercially viable [6]. Potential applications of such memristors span diverse fields ranging from nonvolatile memories on the nano-scale [5,7] to modeling neural networks [8,9]. In the meantime people have observed that all the properties of memristor and the Chua diode can be replicated with the help of some op-amp combination [10,11]. As such many of the studies involving memristor utilize op-amp combination [12,13]. An important field of investigation is coupling of such circuits in different ways [14]. A very significant incident in this respect is the phenomenon of synchronization [15,16]. But, this is crucially depending on the nature of the corresponding coupling. This is immensely important from the point of view of both experiment and theory. Coupling between some variables of two or more non-linear systems leading to synchronization is a well documented fact. This has been observed in many physical [17,18], chemical [19], ecological [20,21] and biological systems [22]. Later this phenomenon found one of its many applications in cryptography and secure communications. Moreover, coupling nonlinear systems in different spatial configurations leads to the construction of spatiotemporal systems that can exhibit a variety of exotic dynamical behaviors such as pattern formation, wave propagation,

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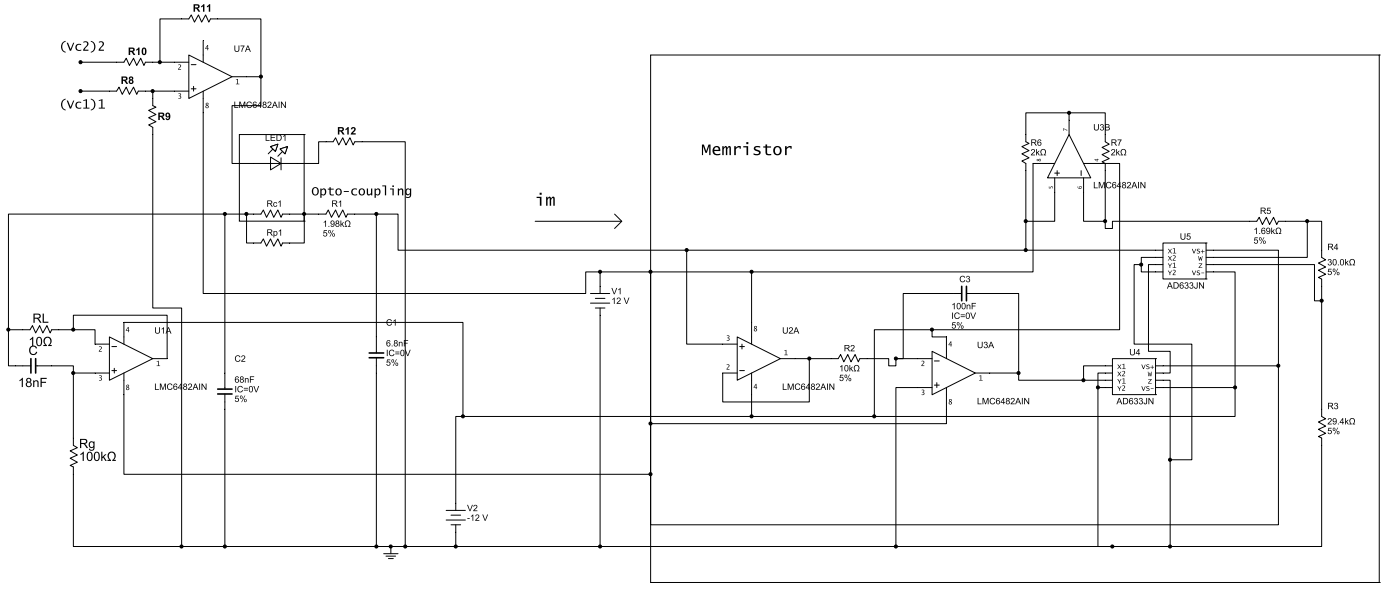

Fig. 1. Single Chua circuit with opto-coupler.

rotating spirals [23] and chimera. They mimic spatiotemporal dynamics, observed in biological systems [24], very well. Finally, recent works have shown that coupling nonlinear elements can invoke a plethora of interesting phenomena, such as hysteresis, phase locking, phase shifting, phase flip, amplitude death [25–27] and oscillation death [28] in the dynamical behavior of the coupled systems. In recent work, the transition between amplitude death and oscillation death [29,28] is also explained from the bifurcation point of view. In the present work, we provide both numerical and experimental results for amplitude death when two systems are coupled bidirectionally by conjugate variables. These involve oscillators exhibiting dynamics such as period one, period three, and chaotic. Furthermore, a new phenomenon involving generation of oscillations has been explored. In this case, the autonomous dynamics of the uncoupled systems exhibit fixed-point behavior. Conjugate coupling makes these fixed points unstable, and consequently both systems start oscillating with either periodic or chaotic dynamics depending on the strength of the coupling term. But in addition to these, we discussed the path to these transition filled with transient effects. Sometimes transient effects make the situation appear much before it occurs. Our results indicate that using the conjugate coupling mechanism one can efficiently regulate the dynamical behavior of the coupled system, making this technique a very powerful tool for controlling nonlinear systems. But, one has to be very careful about leaving the system enough time for its transient to have died down. Before going into the details, we first describe conjugate coupling, amplitude death, rhythmogenesis (generation of oscillation) and inductorless designs of memristive Chua circuit.

When more than two dynamical variables are present in a system, any two non-similar variables are described as conjugate variable. If a coupling is made through the use of such variables then the corresponding coupling is called conjugate coupling. Such couplings are used in experiment where one out put is introduced to another system and vice versa. In the present case, coupling term is proportional to the difference between to conjugate variables each taken from one system at a time. This coupling are introduced to two systems symmetrically and they are coupled through parameters.

In two coupled nonlinear system, amplitude death describes the phenomenon of cessation of oscillation in each individual system through driving themselves towards a stable fixed point. The coupling can be delayed or conjugate and the fixed point can be pre-existing or created through coupling. Systems undergoing this phenomenon can be identical or parameter mismatched. Though, initially an emphasis was given to parameter mismatch and time delayed interaction. But later these conditions were modified. Recently, conjugate coupling was shown to drive coupled systems to oscillation death. As the coupled system goes to a stable fixed point, amplitude death can be indicated through the negative maximum Lyapunov exponent (MLE) of the coupled system. Thus, we have used negative MLE of the coupled system to indicate amplitude death (AD).

In a way, conjugate effect of AD is generation of oscillation through coupling when individual systems are at their respective fixed points. Coupling changes the stability of the fixed point to make the individual systems oscillates with rhythm. This phenomenon is called throughout the paper as rhythmogenesis [30–33].

In paper [11], Muthuswamy described a Chua circuit were the nonlinear resistance is replaced with memristor. They call that circuit as memristive Chua circuit. Though the circuit is easy to use, they used inductance to manufacture the circuit. In most electronic experiments, inductor is a less than desirable elements [34]. They are treated that way for a number of reasons. Inductors are not as standard as other elements in electronic circuits. Most of the time they have to be manufactured according to the need of particular experiment. They are not as ideal as other circuit elements. They are usually large in dimension than any other circuit elements unless they inductance is small. Generally, a high value of inductance is used in memristive Chua circuit. So, we developed a new inductorless realization of Chua circuit (Fig. 1). Here, inductance is replaced with the help of a Wien bridge type oscillator and the modified Chua circuit has same chaotic behavior.

In the present communication we have coupled two Chua circuit optically through a LED/LDR pair (opto-coupler) and have seen that a completely new phenomenon to take place which is called transient AD. On the other hand, the variation of the coupling parameter also leads to transient rhythmogenesis. By “transient AD”, we mean occurrence of AD state which occurs much earlier in time scale and for much higher coupling value. Interestingly, same feature is repeated in its path to rhythmogenesis. “Transient rhythmogenesis” seem to occur for much higher value of coupling and for much earlier in time scale.