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Instability in the magnetic field penetration in type II superconductors

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ABSTRACT

Under the view of the time-dependent Ginzburg–Landau theory we have investigated the penetration of the magnetic field in the type II superconductors. We show that the single vortices, situated along the borderline, between the normal region channel and the superconducting region, can escape to regions still empty of vortices. We show that the origin of this process is the repulsive nature of vortex–vortex interaction, in addition to the non-homogeneous distribution of the vortices along the normal region channel. Using London theory we explain the extra gain of kinetic energy by the vortices situated along this borderline.

1. Introduction

In 1994 a new phenomenon was discovered, the appearance of macro-turbulence in type II superconductors. V.K. Vlasko-Vlasov et al. showed that during the remagnetization process of high-T_c superconductors, in some range of temperature, the vortices condense into drops of increasing density, which then move along the sample. They believed that this process appears only when a noticeable creep takes place [1]. This process is a turbulence relaxation. The same phenomenon was studied by Koblischka et al. [2]. They showed that when a reversed external magnetic field is applied to a remnant state, droplets of vortices are formed and escape from the flux front. These vortices move along the sample. They concluded that, this phenomenon is due to the heat released in the vortex–antivortex annihilation process. In 2009, I.V. Voloshin et al. [3] studied the macro-turbulent instability in a YBCO single crystal. In their experiment the turbulence manifested itself by the advance of the magnetization-reversal front into the interior of the sample. In all the above quoted papers, there are vortices moving along the superconducting region, which leaves the normal region channel inside the sample.

In the present article, a similar effect is investigated. Vortices situated along the borderline, between the normal region channel and superconducting region, can escape to regions that are still empty of vortices. Note that these superconducting regions are devoid of defects, pinning centers, inversion of applied magnetic field, or any other effect. Our aim here is to explain that this phenomenon is produced by the non-homogeneous distribution of the vortices along the normal region channels, and also because of repulsion interaction between vortices. Both effects are strongly related to the size of the sample. Related experimental work was published in Ref. [4], where the authors studied a continuum and also the discrete distribution of the magnetic flux in large mesoscopic superconductor disk.

The textbooks classify superconductors by the ratio of the magnetic penetration depth λ to the coherence length ξ. According to the Ginzburg–Landau (GL) theory the type I and type II superconductivity interchange when the GL parameter $\kappa = \lambda/\xi$ reaches the critical value $\kappa_c = 1/\sqrt{2}$ [5–7]. However, several experimental and theoretical evidences revealed that the interchange occurs along a finite interval around Bogomolny critical point κ_c. The Bogomolny point is infinitely degenerated with respect to vortex spatial configuration [8,9]. Very close to this point a long-range attraction of vortices appears, which is responsible for the new patterns where the Meissner and Abrikosov-lattice domains coexist. It is often referred to as type II/1 state, and recently it was referred to as type 1.5 superconductor [10–15].

For type II superconductors the penetration of magnetic field occurs via quantum of magnetic field and it is possible to enumerate the number of vortices in the sample [16,17]. For large samples, the magnetic field penetration occurs, apparently, as "average" flux density distribution. It can be explained by the Bean model, since in the Bean model the penetration of the magnetic field seems to be continuous [18–22]. New phenomena have been

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appearing for different materials. For instance, dendrites in large sample of MgB_2 [25], coexistence of Meissner and Abrikosov state in small sample of MgB_2 [26], avalanches in Nb film, [27], dendritic flux in YBaCuO films [28], fractional vortex states in two-band mesoscopic superconductors [29–31], the broken-symmetry vortex states, and fractional-flux vortex states with broken time-reversal symmetry [32].

The use of the high-resolution magneto-optical technique has permitted detailed investigation of the magnetic flux penetration in superconductors [33–35]. Magneto-optical images permit to observe that, along of the borderline between the normal region channel and the superconducting region, one can found small magnetic structures. The formation of these structures indicates a new arrangement of vortices along of the normal region channel, and furthermore, it indicates a non-homogeneous distribution of vortices close to the borderline. In order to study the dynamic of magnetic flux penetration, for imaging, one important devise was built by E. Zeldov et al. [36].

In this paper we treat a discrete distribution of vortices. The configuration obtained in this article through TDGL equations can be experimentally verified.

The outline of this paper is as follows. In Section 2 is presented the TDGL model used in this article and also the numerical procedure. In Section 3 is shown some snapshots of the magnetic field penetration which result of our numerical simulations. In Section 4 is presented an approach to explain the jumps of vortices under the view of the London theory. A conclusion is given in Section 5.

2. Theory and numerical simulation

The superconducting state can be represented by the complex order parameter $\Psi(x, y, z, t)$, where $|\Psi|^2$ is the density of superconductors electrons at position (x, y, z) at time t. The superconductivity is suppressed at the center of the vortex where $|\Psi|^2 = 0$, where the local magnetic field, $\mathbf{B}$ is maximum. The Gibbs energy of the superconducting state is $G_s = G_n - \alpha|\Psi|^2 + (\beta/2)|\Psi|^4$, where G_n is the Gibbs energy of the normal conducting state. The parameter β is assumed constant and α is related to the temperature through $\alpha(T) = \alpha(0)(1 - T/T_c)$. For a type II superconductor the time-dependent Ginzburg–Landau (TDGL) equations, coupled to a penetrating magnetic field $\mathbf{B}_a = \nabla \times \mathbf{A}$, with $\mathbf{A}$ being the magnetic vector potential, reads in SI units [37]

$$\frac{\hbar^2}{2mD}\left(\frac{\partial}{\partial t} + i\frac{q}{\hbar}\Phi\right)\Psi = -\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2\Psi + \alpha\Psi - \beta|\Psi|^2\Psi, \quad (1)$$

$$\sigma\left(\frac{\partial \mathbf{A}}{\partial t} + \nabla\Phi\right) = \frac{q\hbar}{2mi}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - \frac{q^2}{m}|\Psi|^2\mathbf{A} - \frac{1}{\mu_0}\nabla\times\nabla\times\mathbf{A}. \quad (2)$$

The charge of a Cooper pair is denoted $q = 2e$ and the Cooper pair mass is m. The parameter D is a phenomenological diffusion coefficient [38]. An important property of the time-dependent model is that the current density $\mathbf{J}$ is $\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}_s$, where $\mathbf{J}_s$ is the supercurrent, and σ is the coefficient of normal conductivity. Here $\Phi = \Phi(x, y, z, t)$ is the electric potential. $\mathbf{E}$ is the electric field given by $\mathbf{E} = -\partial\mathbf{A}/\partial t - \nabla\Phi$. The parameter μ_0 is the permeability of the free space. The model is scaled with respect to λ and with the dimensionless variables marked by primes, Eqs. (1) and (2) can be transformed through [39]

$$(x, y, z, t) = (\lambda x', \lambda y', \lambda z', \frac{\xi^2}{D}t'),$$

$$\mathbf{A} = \frac{\hbar}{q\xi}\mathbf{A}',$$

$$\Psi = \sqrt{\frac{\alpha}{\beta}}\Psi',$$

$$\Phi = \alpha D\kappa^2\sqrt{\frac{2\mu_0}{\beta}}\Phi',$$

$$\sigma = \frac{1}{\mu_0 D\kappa^2}\sigma'. \quad (3)$$

After some algebra and omitting the prime on the dimensionless variables, the Ginzburg–Landau equations can be rewriten as

$$\left(\frac{\partial}{\partial t} + i\kappa\Phi\right)\Psi = -\left(\frac{i}{\kappa}\nabla + \mathbf{A}\right)^2\Psi + (1 - T)\Psi\left(1 - |\Psi|^2\right), \quad (4)$$

$$\sigma\left(\frac{\partial \mathbf{A}}{\partial t} + \nabla\Phi\right) = \frac{1}{2i\kappa}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - |\Psi|^2\mathbf{A} - \nabla\times\nabla\times\mathbf{A}, \quad (5)$$

where T is the reduced temperature ($T = T/T_c$), $\kappa = \lambda/\xi$ is the Ginzburg–Landau constant, where $\xi = \hbar/\sqrt{(2m\alpha)}$ is the coherence length. Since that TDGL equations, (4) and (5), are gauge invariant under the transformations [37] $\tilde{\Psi} = \Psi\exp(i\chi)$, $\tilde{\mathbf{A}} = \mathbf{A} + \nabla\chi$, and $\tilde{\Phi} = \Phi - \partial\chi/\partial t$, where $\chi(x, y, z, t)$, and using the zero-scalar potential gauge, $\Phi = 0$, at all times and positions, the TDGL equations above can be rewriten as

$$\frac{\partial\Psi}{\partial t} = -\left(\frac{i}{\kappa}\nabla + \mathbf{A}\right)^2\Psi + (1 - T)\Psi\left(1 - |\Psi|^2\right), \quad (6)$$

$$\frac{1}{\sigma}\frac{\partial \mathbf{A}}{\partial t} = \frac{1}{2i\kappa}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - |\Psi|^2\mathbf{A} - \nabla\times\nabla\times\mathbf{A}. \quad (7)$$

In order to solve the TDGL equations (6) and (7), we need to specify the boundary conditions of the superconducting sample, which are

$$(-i\nabla - \mathbf{A})\Psi\cdot\mathbf{n} = 0, \quad (8)$$

$$\nabla\times\mathbf{A} = \mathbf{B_a}, \quad (9)$$

$$\mathbf{E}\cdot\mathbf{n} = 0. \quad (10)$$

These conditions are the usual boundaries conditions, where the first one is related to the density of current found only inside the superconducting sample and the second one is related to the continuity of the magnetic field. To justify the third boundary condition, we must remember that the total density of current of superconducting sample is given by $\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}_s$, where $\mathbf{J}_s$ is the density of supercurrent and $\mathbf{E}$ is the electrical field. As the normal current is parallel to the electrical field and as the current does not pass across the boundary of the superconductor, it is easy to see that the normal component of the electrical field must be zero on the boundary. The last boundary condition, Eq. (10), can be rewritten as

$$\left(\frac{\partial \mathbf{A}}{\partial t} + \nabla\Phi\right)\cdot\mathbf{n} = 0. \quad (11)$$

Recently Alstrom et al. [39] solved numerically the TDGL equations for various bidimensional geometries. They had used the boundary conditions, $\nabla\Psi\cdot\mathbf{n} = 0$, $\nabla\times\mathbf{A} = \mathbf{B_a}$, and $\mathbf{A}\cdot\mathbf{n} = 0$. The last one is a consequence of the gauge invariance chosen ($\Phi = 0$), which permits writing $\partial\mathbf{A}/\partial t\cdot\mathbf{n} = 0$, and after the integration it gives us $\mathbf{A}\cdot\mathbf{n} = \text{const}$. They had assumed this constant as zero. Their