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Agent-based financial dynamics model from stochastic interacting epidemic system and complexity analysis

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ABSTRACT

An agent-based financial stock price model is developed and investigated by a stochastic interacting epidemic system, which is one of the statistical physics systems and has been used to model the spread of an epidemic or a forest fire. Numerical and statistical analysis are performed on the simulated returns of the proposed financial model. Complexity properties of the financial time series are explored by calculating the correlation dimension and using the modified multiscale entropy method. In order to verify the rationality of the financial model, the real stock market indexes, Shanghai Composite Index and Shenzhen Component Index, are studied in comparison with the simulation data of the proposed model for the different infectiousness parameters. The empirical research reveals that this financial model can reproduce some important features of the real stock markets.

1. Introduction

Recently, the study of statistical and complexity behaviors of the market fluctuations has been a focus of economic research for a more clear understanding of the financial market dynamics. With the financial markets deregulated by the governments all over the world, the modeling of the dynamics of the forward prices is becoming a key problem that should be highly focused on in the risk management, derivatives pricing and physical assets valuation [11,42,43]. Due to the fact that a financial market can be considered as a strongly fluctuating complex system with the large numbers of interacting factors, various dynamic models have been introduced into the financial system in an attempt to reproduce and analyze the fluctuations of the stock markets, and most of the models are based on the perspective that the changes of the prices are caused primarily by the transmission of new information or the idea that the price fluctuations are due to the interaction among the market investors, see [16,17,22,23,28,30,32,35,36,41]. For example, Lux and Marchesi [22] introduced an agent-based model in which chartist agents compete with fundamentalists agents, leading to power-law distributed returns as observed in real markets which contradicts the popular efficient market hypothesis. Stanley et al. [28] pointed that financial markets are similar to physical systems in that they are comprised of a large number of interacting agents, and many statistical physics systems or interacting particle systems [4,10,15,19–21,26,27,34,39–41] have been applied to study the fluctuation behaviors and the complexity of financial markets. For instant, Zhang and Wang [41] invented an interacting-agent model of speculative activity explaining price formation in financial market that is based on the finite-range contact particle system, where the epidemic spreading of the contact model is considered as the dissemination of the investing information in the stock market. In this work, we presents a new financial stock price model for the simulation of financial time series by applying the interacting epidemic model [3,9,24,25].

Financial market is a complex system which has many non-stationary influence factors and its price changes often exhibit important statistical behaviors, such as the fat tails distribution, the power law of logarithmic return and volume, the volatility clustering, the multifractality of volatility, etc. Therefore, it is greatly essential to study the fluctuation behaviors and the complexity properties of the return time series. In the present paper, we first study the basic statistical behaviors of the simulative returns of the proposed financial model. Then we analyze the complexity of the time series by calculating the correlation dimension [8,12,13,27] and using the modified multiscale entropy (MMSE) method [38], which is more effective in complexity quantification of the short-term time series. To make a clear trial comparison, the closing prices of each trading day of Shanghai Stock Exchange Composite Index (SSE) and Shenzhen Stock Exchange Component Index (SZSE) are selected as the actual data. They are both from 25 April, 2001 to 12 September, 2013, and the total numbers of them are about 3001 respectively.

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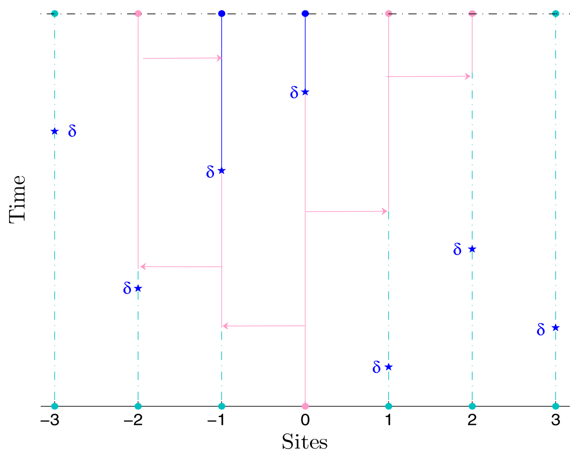

Fig. 1. Graphical illustration of the mechanism to simulate the one-dimensional interacting epidemic model.

2. Financial price model by the stochastic interacting epidemic system

2.1. Description of the stochastic interacting epidemic model

In this section, we give a brief description of the one-dimensional epidemic model (or stochastic interacting epidemic system) in epidemic language, for details see Section 9 of [9]. In this epidemic model, each size $x \in \mathbb{Z}$ can be in one of the three states: 1, i, or 0; and the state of the process is represented by a function $\xi_t : \mathbb{Z} \to \{1, i, 0\}$, $\xi_t(x)$ giving the state of x at time t. In the epidemic interpretation, 1 = susceptible, i = infected, 0 = immune. An infected individual emits germs according to a Poisson process with rate λ. A germ emitted from x goes to one of its two neighbors $x+1$, $x-1$ with equal probability. If the germ goes to the site of a susceptible individual, then that individual immediately becomes infected and begins to emit germs. He stays infected for a random amount of time with distribution $\mathbb{F}$, then recovers and is immune from further infection according to a Poisson process with rate υ (we consider $\upsilon = 1$ in this paper). In formulating the dynamics, we can think of a disease like measles where recovered individuals (if it is the infected one) cannot get the disease again. We declare that the infection periods and the Poisson process of germs associated with different sites are independent.

Given the above description, we introduce the construction process of the one-dimensional epidemic model with the state space $\{1, i, 0\}^{\mathbb{Z}}$ on the time space $\mathbb{Z} \times \mathbb{R}_+$, to illustrate the simulation of the epidemic model. For each pair $x, \tilde{x} \in \mathbb{Z}$ with $|x - \tilde{x}| = 1$, let $\{U_n^{(x,\tilde{x})}, n \geq 1\}$ and $\{T_n^{(x)}, n \geq 1\}$ be independent Poisson processes with rate λ and υ respectively. $\{U_n^{(x,\tilde{x})}, n \geq 1\}$ indicates the moment when x (if it is the infected one) infects $\tilde{x}$ (if it is the susceptible one), and $\{T_n^{(x)}, n \geq 1\}$ represents the timing when x (if it is infected) turns into the immune and cannot be infected again. Fig. 1 displays a graphical representation of the epidemic model, where a sign 'δ' is placed at the Poisson point in the process $\{T_n^{(x)}, n \geq 1\}$, representing the immunity timing. At the initial time, the individual at origin is infected by a disease while others are susceptible healthy people. With time going, the states of individuals change gradually according to the epidemic mechanism. Different colors are used to distinguish their states in Fig. 1.

2.2. Establishment of the financial price model

Throughout this section, we model the financial price and the return process of a stock market by using the epidemic model, which is composed of a large number of interacting "agents" and has similarity with the financial markets. The motivation of modeling stock prices by the epidemic model is to uncover the empirical laws in the real stock markets and have a better understanding of the dynamics of the financial systems.

In this paper, we assume that the investors' investment decisions towards the financial market lead to the fluctuations of the stock prices, and the investment attitudes can be influenced by the news spreading in the stock markets. Suppose that each trader can trade the stock several times at each day $t \in \{1, 2, \dots, T\}$, but at most unit number of the stock at each time. Let l be the time length of the trading day, we denote the stock price at time s in the tth trading day by $\mathscr{P}_t(s)$, where $s \in [0, l]$. We assume that the stock market consists of $M+1$ (M is large enough) traders, who are located in a line $\{-M/2, \dots, -1, 0, 1, \dots, M/2\} \subset \mathbb{Z}$. At the very beginning of each trading day, we select a certain proportion of traders (with the initial distribution p_0) randomly in the system as infected individuals in the epidemic model, marked as type-inf, representing those who receive some market news that will affect their trading decisions, and leave the other traders as type-sus (corresponding to the susceptible individuals or healthy individuals). We define a random variable $\zeta(t)$ of values $1, -1, 0$ to represent the buying position, selling position and neutral position of type-inf investors with probabilities p, q and $1-p-q$ respectively. According to the epidemic process, the investors can affect each other or the news can be spread, which is considered as the main factor of the price fluctuations. Specifically, the investors in type-inf can spread the market news to their neighbors in type-sus. When the investors in type-inf change into type-imm, it will mean that the news becomes invalid for them and their attitudes cannot be altered by this news again, just like the case of site '−1' in Fig. 1. For the case of site '1', the investor in type-sus cannot change into type-imm because of the vacancy of the news at the initial time, but he can change into type-inf when he gets the news from his left-side neighbor at site '0' and then he can also infect or spread the news to his right-side type-sus neighbor at site '2'. We mark the numbers of type-sus investors, type-inf investors, and type-imm investors at time t as $N_t^{sus}(s)$, $N_t^{inf}(s)$, and $N_t^{imm}(s)$ respectively. Then the aggregate excess demand for the returns at time t $(t = 1, 2, \cdots, T)$ is given by

$$\mathscr{W}_t(s) = \zeta(t)\left[\eta_1 N_t^{inf}(s) - \eta_2 N_t^{imm}(s)\right] \Big/ (M+1), \quad s \in [0, l] \tag{1}$$

where $M+1 = N_t^{sus}(s) + N_t^{inf}(s) + N_t^{imm}(s)$, η_1 and η_2 are the impact factors for the type-inf investors and type-imm investors respectively. From the above definitions and [2,18,31], the formula of a discrete time stock price is defined as

$$\mathscr{P}_t(s) = \mathscr{P}_{t-1}(s) \exp\{\gamma \mathscr{W}_t(s)\},$$
$$\mathscr{P}_t(s) = \mathscr{P}_0 \exp\left\{\sum_{k=1}^{t} \gamma \mathscr{W}_k(s)\right\} \tag{2}$$

where $\gamma > 0$ is the depth parameter of the market which measures the sensitivity of stock price fluctuations to the change in excess demand, and $\mathscr{P}_0$ is the initial stock price at trading day $t = 0$. The formula of the stock logarithmic return [30,41] is followed as

$$r_t = \ln \mathscr{P}_t - \ln \mathscr{P}_{t-1}, \quad t = 1, 2, \cdots, T. \tag{3}$$

In the present paper, we will focus on the statistical fluctuation properties of the returns with four different parameters in the epidemic model, that is, the transmission rate λ, the initial distribution of infected patients p_0, the impact factor η_1 for type-inf individual and the impact factor η_2 for type-imm individual. The simulated data will be different as long as the combination values of these four parameters change. In Fig. 2, the fluctuations of