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Controllable resonant tunnelling through single-point potentials: A point triode

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ABSTRACT

A zero-thickness limit of three-layer heterostructures under two bias voltages applied externally, where one of which is supposed to be a gate parameter, is studied. As a result, an effect of controllable resonant tunnelling of electrons through single-point potentials is shown to exist. Therefore the limiting structure may be termed a "point triode" and considered in the theory of point interactions as a new object. The simple limiting analytical expressions adequately describe the resonant behaviour in the transistor with realistic parameter values and thus one can conclude that the zero-range limit of multi-layer structures may be used in fabricating nanodevices. The difference between the resonant tunnelling across single-point potentials and the Fabry–Pérot interference effect is also emphasized.

1. Introduction

The models with zero-range potentials that describe point or contact interactions (see book [1] for details and references) are widely used in various applications to physics. Currently, because of the rapid progress in fabricating nanoscale quantum devices, of particular importance is the point modelling of different extra thin structures like quantum waveguides [2,3] or spectral filters [4]. The point interactions in higher dimensions [5–8] and their nonlinear generalizations [9–11] have been of considerable study as well. These models admit exact closed analytical solutions and they provide relative simple situations, where an appropriate regularization procedure can be chosen to be in relevance with real structures being important for technological applications.

In the simplest case, the point interactions are described by one-dimensional Schrödinger operators with singular zero-range potentials given in the form of distributions. Within the distributional approach, these operators have been shown to exhibit peculiar features under certain conditions [12–17]. Surprisingly, the potentials in the form of the derivative of Dirac's delta function appear to be fully non-transparent for almost all values of the strength constant, except for a discrete (resonance) set at which the transmission occurs non-zero [18–24]. Moreover, as shown recently [25], a whole family of point potentials with full transmission at the resonance set can be constructed as a zero-range limit of three- or four-layer heterostructures. Therefore it would be of interest in applying the effect of one-point resonant tunnelling to the design of different types of nanodevices with adding potentially-controllable parameters.

In this Letter, we exploit a simple idea that suggests how to control the electron flow across the potentials located at a single point. Consider the Hamiltonian $\mathcal{H} = -d^2/dx^2 + \alpha D(x)$ where $x \in \mathbb{R}$ is a spatial variable, with a strength constant α and a distribution $D(x)$. Assume next that this distribution has non-zero "positive" and "negative" parts in the sense that any its regularization can be represented by a sequence $\Delta_\varepsilon(x) = \Delta_\varepsilon^+(x) + \Delta_\varepsilon^-(x)$ with $\Delta_\varepsilon^+(x) \geq 0$ (a "barrier") and $\Delta_\varepsilon^-(x) \leq 0$ (a "well") where ε is a squeezing parameter. In this case, both the norm convergence of the renormalized Hamiltonians $\mathcal{H}_\varepsilon = -d^2/dx^2 + \alpha\Delta_\varepsilon(x)$ and the distributional limit $\Delta_\varepsilon(x) \to D(x)$ as $\varepsilon \to 0$ can rigorously be defined. However, from a practical point of view it is of interest to introduce two strength constants, one of which is being responsible for the positive part of the distribution and the other one for the negative part. In physical terms, the coefficient at the positive part can be considered as a "system" or a "structural" parameter being fixed in the model under investigation, whereas the other coefficient may be used as a varying "controllable" ("gate") parameter. In this regard, some distributions like the derivative of Dirac's delta function $\delta'(x)$, can be obtained in the zero-range limit from a sequence of regular functions $\Delta_\varepsilon(x)$ for which $\int_{\mathbb{R}} \Delta_\varepsilon(x)dx = 0$. The key point of our approach is to replace the sequence $\alpha\Delta_\varepsilon(x)$ by $\alpha^+\Delta_\varepsilon^+(x) + \alpha^-\Delta_\varepsilon^-(x)$ with two positive strength constants α^+ and α^-. Clearly, it is not possible in general to define a distributional limit for this modified sequence if $\alpha^+ \neq \alpha^-$, however, the zero-range limit of the regularized Hamil-

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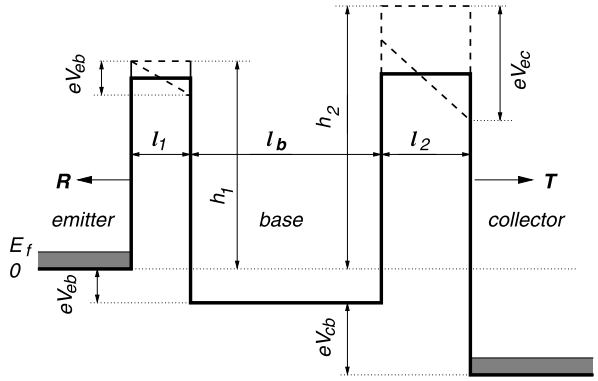

Fig. 1. Schematic energy diagram of a three-layer heterostructure with two applied potentials eV_{eb} and eV_{cb}. Positive polarities are shown and, instead of tilted potentials, averaged flat steps assumed. The solid line shows potential (2) with $\varepsilon_1 = \varepsilon_b = \varepsilon_2 = 1$. Horizontal dashed lines correspond to the structure without applied voltages.

tonians $-d^2/dx^2 + \alpha^+\Delta_\varepsilon(x) + \alpha^-\Delta_\varepsilon(x)$ may admit under appropriate conditions a well-defined self-adjoint operator in the $\varepsilon \to 0$ limit. In this way, one can essentially enlarge the family of point interactions with the application to three-terminal quantum devices. As we shall see below, the constants $\alpha^\pm$ can be expressed in terms of voltages applied externally to a multi-layer heterostructure. Within a scattering approach, varying one of these voltages and therefore the constants $\alpha^\pm$, one can control the transmission properties of an electron flow across a heterostructure.

As a rule, a heterostructure is composed of plane layers, where the electron motion is confined in the longitudinal x direction being free in the transverse direction. In simple cases, the three-dimensional Schrödinger equation of such a structure can be separated into longitudinal and transverse parts, writing the total electron energy E as the sum of the longitudinal and transverse energies: $E = E_l + \hbar^2 k_t^2/2m^*$, where m^* is an effective electron mass, and expressing the wave function by the product $\psi = \psi_l \psi_t$. As a result, we arrive at the reduced one-dimensional Schrödinger equation with respect to the longitudinal components of the wave function $\psi_l(x)$ and the electron energy E_l. For the brevity of notations, in the following we shall omit the subscript "l" at both $\psi_l(x)$ and E_l.

Next, having applied voltages which are necessary for the appearance of a net tunnelling current through a heterostructure, we need to introduce such objects as an external source of electrons (an emitter or a cathode) and an electron drain (a collector or an anode). When additionally a base or a grid is embedded in a heterostructure and a controllable voltage relative to the emitter is applied, this structure works as a three-terminal device, i.e., a transistor.

In this Letter, we consider a typical transistor (see, e.g., [26–28]), the energy diagram of which is depicted in Fig. 1, and pose the question what is an exactly solvable model of this device in the limit when its dimensions are confined to a point. Therefore it is reasonable to term such a zero-thickness limit as a "point triode". As illustrated by Fig. 1, the structural parameters of the double-barrier transistor are given by heights h_1 and h_2, and widths l_1 and l_2. The applied voltages are V_{eb} (emitter-base, gate) and V_{cb} (collector-base), so that the total bias potential across this three-terminal device becomes $\mathcal{V} \doteq eV_{ec}$, where $-e$ is the electronic charge and $V_{ec} = V_{eb} + V_{cb}$ is the emitter–collector voltage.

2. The reflection-transmission coefficients for a three-layer heterostructure

With a number of simplified assumptions (such as an effective electron mass m^*, being the same in the emitter, base and collector regions, a separable wave function implying free motion in the perpendicular plane plus confined motion in the x direction, a flat-step approximation of applied voltages as shown in Fig. 1, etc.), the reduced one-dimensional Schrödinger equation reads

$$-\psi''(x) + U_\varepsilon(x)\psi(x) = E\psi(x), \tag{1}$$

where the prime stands for the differentiation with respect to the spatial coordinate x, $\psi(x)$ and E are the longitudinal components of the wave function and the electron energy, respectively. Here and in the following, we use the units in which $\hbar^2/2m^* = 1$. The potential $U_\varepsilon(x)$, defined within the heterostructure region, reads

$$U_\varepsilon(x) = \begin{cases} (h_1 - eV_{eb}/2)\varepsilon_1^{-2} \\ \quad \text{for } 0 < x < \varepsilon_1 l_1, \\ eV_{eb}\varepsilon_b^{-2} \\ \quad \text{for } \varepsilon_1 l_1 < x < \varepsilon_1 l_1 + \varepsilon_b l_b, \\ (h_2 - eV_{eb} - eV_{cb}/2)\varepsilon_2^{-2} \\ \quad \text{for } \varepsilon_1 l_1 + \varepsilon_b l_b < x < L_\varepsilon, \end{cases} \tag{2}$$

where the three-component vector $\varepsilon = (\varepsilon_1, \varepsilon_b, \varepsilon_2)$ is a three-scale squeezing parameter and $L_\varepsilon \doteq \varepsilon_1 l_1 + \varepsilon_b l_b + \varepsilon_2 l_2$ is the device thickness that tends to zero as $\varepsilon \to 0$. Schematically, potential (2) with $\varepsilon = (1, 1, 1)$ is shown in Fig. 1 by solid line.

The solution of Eq. (1) with potential (2) can be given in terms of the transfer matrix Λ_ε that connects the boundary conditions for the function $\psi(x)$ and its derivative $\psi'(x)$ at $x = 0$ and $x = L_\varepsilon$:

$$\begin{pmatrix} \psi(L_\varepsilon) \\ \psi'(L_\varepsilon) \end{pmatrix} = \Lambda_\varepsilon \begin{pmatrix} \psi(0) \\ \psi'(0) \end{pmatrix}, \quad \Lambda_\varepsilon = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}, \tag{3}$$

with the elements

$$\begin{aligned} \lambda_{11} &= \cos A_1 \cos B \cos A_2 - (k_1/k_b)\sin A_1 \sin B \cos A_2 \\ &\quad - (k_1/k_2)\sin A_1 \cos B \sin A_2 - (k_b/k_2)\cos A_1 \sin B \sin A_2, \\ \lambda_{12} &= k_1^{-1}\sin A_1 \cos B \cos A_2 + k_b^{-1}\cos A_1 \sin B \cos A_2 \\ &\quad + k_2^{-1}\cos A_1 \cos B \sin A_2 - (k_b/k_1k_2)\sin A_1 \sin B \sin A_2, \\ \lambda_{21} &= -k_1 \sin A_1 \cos B \cos A_2 - k_b \cos A_1 \sin B \cos A_2 \\ &\quad - k_2 \cos A_1 \cos B \sin A_2 + (k_1k_2/k_b)\sin A_1 \sin B \sin A_2, \\ \lambda_{22} &= \cos A_1 \cos B \cos A_2 - (k_b/k_1)\sin A_1 \sin B \cos A_2 \\ &\quad - (k_2/k_1)\sin A_1 \cos B \sin A_2 - (k_2/k_b)\cos A_1 \sin B \sin A_2. \end{aligned} \tag{4}$$

Here, the matrix elements, satisfying the relation $\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} = 1$, are given in terms of

$$\begin{aligned} k_1 &= \sqrt{E - (h_1 - eV_{eb}/2)\varepsilon_1^{-2}}, \qquad k_b = \sqrt{E + eV_{eb}\varepsilon_b^{-2}}, \\ k_2 &= \sqrt{E - (h_2 - eV_{eb} - eV_{cb}/2)\varepsilon_2^{-2}}, \end{aligned} \tag{5}$$

$A_j = k_j l_j \varepsilon_j$, $j = 1, 2$, and $B = k_b l_b \varepsilon_b$. The dependence of matrix elements (4) on ε is given through Eqs. (5).

Beyond the heterostructure region the wave function $\psi(x)$, satisfying Eq. (1), is given by

$$\psi(x) = \begin{cases} \mathrm{e}^{\mathrm{i}kx} + R_\varepsilon\, \mathrm{e}^{-\mathrm{i}kx} & \text{for } -\infty < x < 0, \\ T_\varepsilon\, \mathrm{e}^{\mathrm{i}\gamma kx} & \text{for } L_\varepsilon < x < \infty, \end{cases} \tag{6}$$

where R_ε and T_ε are the reflection and transmission coefficients, respectively, $k \doteq \sqrt{E}$ is the longitudinal wave vector, and $\gamma = \sqrt{1 + eV_{ec}/E}$. The amplitudes of reflection ($\mathcal{R}_\varepsilon \doteq |R_\varepsilon|^2$) and transmission ($\mathcal{T}_\varepsilon \doteq \gamma|T_\varepsilon|^2$) can be given in terms of elements (4) as follows