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Maximum entropy networks are more controllable than preferential attachment networks

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ABSTRACT

A maximum entropy (ME) method to generate typical scale-free networks has been recently introduced. We investigate the controllability of ME networks and Barabási–Albert preferential attachment networks. Our experimental results show that ME networks are significantly more easily controlled than BA networks of the same size and the same degree distribution. Moreover, the control profiles are used to provide insight into control properties of both classes of network. We identify and classify the driver nodes and analyze the connectivity of their neighbors. We find that driver nodes in ME networks have fewer mutual neighbors and that their neighbors have lower average degree. We conclude that the properties of the neighbors of driver node sensitively affect the network controllability. Hence, subtle and important structural differences exist between BA networks and typical scale-free networks of the same degree distribution.

1. Introduction

The scale-free character of complex networks has recently been the focus of much attention. Many models to generate scale-free networks have been proposed [1–5], the most famous of which is the Barabási–Albert (BA) model of preferential attachment [1]. Preferential attachment means that new nodes attach preferentially to the (usually older) nodes with higher degree. To accurately represent the much wider spectrum of real networks, many modified preferential attachment mechanisms have been introduced, including nonlinear preferential attachment [6] and local preferential attachment [7,8]. In addition, many alternative mechanisms for preferential attachment have been proposed in recent years [9,10]. Goh et al. [11] use the load exponent to construct networks with different values of γ, which is valid for both undirected and directed cases. Park et al. [12,13] propose a self-organizing mechanism to generate scale-free networks. Despite this proliferation of scale-free network models, it remains unclear how likely (in a probabilistic sense) are realizations of each model [14–16].

Since the advent of various algorithms to generate scale-free networks, one of the most closely examined properties of such networks has been their controllability. The venerable field of control has found fertile new ground in examining the controllability of large dynamical systems that can be described by networks of simpler components. A system (here a network of coupled low-dimensional dynamical systems) is said to be controllable if it can be driven from an arbitrary initial condition to an arbitrary final state in finite time with bounded perturbations. Controllability of complex networks is an issue of primary importance in various fields, and has attracted much interest in recent years [17–20]. As a combination of control theory and network science, exploring network controllability can deepen our understanding of dynamics of complex systems, and ultimately enable us to control them. While preferential attachment persists as a default generative proxy for scale-free networks, we find that the controllability of such networks is much worse than what one we would expect for most (connected) networks with a given power-law degree distribution.

While many phenomena have been attributed to the scale-free nature of network degree distribution, it is not always clear to what extent these phenomena are properties generic to networks with a power-law degree distribution, or, whether these properties are more peculiar to a particular network generation model. That is, for a given degree distribution, one can construct many different networks conforming to that degree distribution. The same is true in particular for the many generative growth models. It is possible to grow a network converging to a given degree distribution in many different ways — and often one achieves quite different classes of networks. This is true, in particular, for power-law degree distributions. Because of the particular enthusiasm with

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which scale-free networks have been studied it is important to recognize exactly which networks one is describing [21]. Many networks share the same degree distribution and it is clear that degree distribution alone is insufficient to characterize network structure. Other properties also play an important role in the function and dynamics of networks [15] — a trivial example of this is the minimum degree. Consequently, we cannot ensure that particular network properties are due to the scale-free property alone.

In the next section we introduce our computational algorithm. In Section 3 we present our results and in Section 4 we conclude.

2. Algorithm

Deciding how to randomly generate a *typical* scale-free network with an arbitrary given degree distribution is a necessary first step in addressing this problem. By *typical* we mean a randomly chosen network from the space of all networks conforming to our particular family of scale-free networks. Hence, we need to define that family: in particular, we restrict our interest to connected networks with no self-loops or multiple edges. If a randomly chosen network with a particular degree distribution also has these properties, we call it *viable*. In the remainder of this paper our primary interest is in networks that satisfy each of these constraints. Aside from being viable, a typical network also needs to have two other properties: the probability (or likelihood) of such a network occurring is large, and the network is both random and unbiased. The BA algorithm meets the first two properties, but it does not achieve the third [21]: the BA algorithm is biased. Low (and negative) assortativity in the BA network indicates the nature of this bias. Hubs are necessarily interconnected because these will always grow from the initial seed network — a *rich club* is ensured by the growth process [22]. Because the growth process is (almost always) terminated after a finite time, the last-added low-degree nodes have an excessive likelihood of being connected to the biggest hubs. These are properties peculiar to the BA growth method, and not the prescribed degree distribution.

To address these constraints, a maximum entropy method has recently been proposed [15]. This method applies a standard Markov-Chain Monte Carlo approach to sample the given degree distribution. Although this method is efficient for small networks (an in that case has been shown to converge to a maximum entropy sampling of the space of possible networks), an easier alternative algorithm has been proposed by adding the constraint that the given degree histogram is constant (i.e. one first samples the histogram and then performs network perturbations over that fixed histogram) [23]. This algorithm is divided to two steps: First, generate a viable network with the given degree distribution; Second, apply a MCMC process with only the edge-switching algorithm shown in Fig. 1. They provides us with an efficient approach to generate a typical scale-free network with a given degree histogram. This simple computational expedient of first sampling the degree histogram and then apply edge-switching (while ensuring continued connectivity) and an MCMC process to sample a graph with a fixed histogram, provides equivalent results. Asymptotically the results of this approach converge to the original algorithm [15]. Because these typical scale-free networks are generated by a maximum entropy method [24,16], we henceforth refer to this as a maximum entropy (ME) network.

The experiments we report here are based on a direct comparison between the BA network generated via the preferential attachment algorithm [1] and a ME network with the same degree distribution (to constrain our ME networks to be equivalent to the BA realizations, we use the degree distribution of the BA network when generating the ME realization). Our attention is restricted to BA networks as this is the most widely used algorithm and provides a useful baseline for comparison. First, we use the preferential attachment algorithm to generate an undirected scale-free network (BA), which is used as an initial network. Second, an edge-switching algorithm [25–28] (shown in Fig. 1(a)) is performed on that initial seed network to generate a random viable network with exactly the same degree sequence.

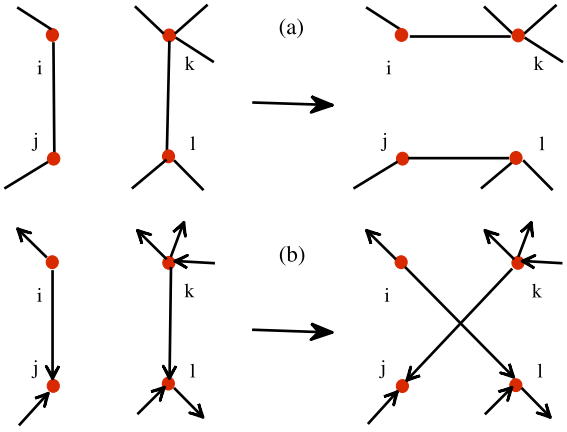

Fig. 1. The example of edge-switching [25]: (a) undirected network, and (b) directed network.

To be precise, the edge-switching algorithm proceeds as follows. Choose four distinct nodes i, j, k and l, and let A be the adjacency matrix, such that $A(i,j) = A(j,i) = 1$, $A(k,l) = A(l,k) = 1$, $A(i,k) = A(k,i) = 0$, $A(j,l) = A(l,j) = 0$, and then switch the links to $A(i,j) = A(j,i) = 0$, $A(k,l) = A(l,k) = 0$, $A(i,k) = A(k,i) = 1$, $A(j,l) = A(l,j) = 1$. To guarantee that the network is connected, we check the network connectivity after each edge-switching step. Only edge-switches that preserve connectivity are permitted. Hence, after sufficiently many edge-switching steps, we obtain a corresponding random scale-free network (ME). Finally, the first and second processes are repeated to obtain a sufficiently large sample of BA–ME pairs.

When considering network control, it is usual to also consider directed networks. While the algorithm proposed in Refs. [15,21] generates undirected ME networks, generating directed networks is a little more complicated. First, we use the preferential attachment algorithm to generate two distinct undirected scale-free networks, G1 and G2. We number the nodes in both G1 and G2 randomly. The link direction of G1 is assigned from smaller label index to larger while G2 assigns links moving from the larger label index to the smaller. Following these index labels, the networks G1 and G2 are combined to one network (that is, the node labeled i in G1 and the node labeled i in G2 are combined into a single node). Second, a similar edge-switching algorithm (Fig. 1(b)) is performed on the network to generate a random viable network while preserving node in-degree and out-degree. Let A be the adjacency matrix, the edge-switching algorithm chooses four distinct nodes i, j, k and l, such that $A(i,j) = 1$, $A(k,l) = 1$, $A(i,l) = 0$, $A(k,j) = 0$, and then switch the links to $A(i,j) = 0$, $A(k,l) = 0$, $A(i,l) = 1$, $A(k,j) = 1$. Last, repeat the first two steps to get enough directed BA–ME pairs.

Now, we can move on to our central question, whether the ME network is more or less controllable than BA and if so, why? From this we can conclude which properties of controllability are inherent to the scale-free degree distribution and which are peculiar to the BA growth model. Controllability of complex networks is an issue of primary importance in various fields, and has attracted much interests in recent years. The most popular method for linear dynamical systems is Kalman's controllability matrix [29, 30], but this requires a brute-force search to identify the minimum number of driver nodes, which makes it difficult to adapt to large networks and complex systems. Therefore, a general and more practical framework for controllability of complex directed