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Characterization of dynamical systems under noise using recurrence networks: Application to simulated and EEG data

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ABSTRACT

In this letter, we study the influence of observational noise on recurrence network (RN) measures, the global clustering coefficient (C) and average path length (L) using the Rössler system and propose the application of RN measures to analyze the structural properties of electroencephalographic (EEG) data. We find that for an appropriate recurrence rate ($RR > 0.02$) the influence of noise on C can be minimized while L is independent of RR for increasing levels of noise. Indications of structural complexity were found for healthy EEG, but to a lesser extent than epileptic EEG. Furthermore, C performed better than L in case of epileptic EEG. Our results show that RN measures can provide insights into the structural properties of EEG in normal and pathological states.

1. Introduction

Many natural systems are inherently governed by nonlinear dynamics. For example, the dynamical behavior of individual neurons in the brain is governed by threshold and saturation phenomena, which give rise to nonlinearity [1]. Thus, nonlinear time series analysis is an important tool in understanding the dynamical properties of the brain using electroencephalography (EEG), which provides temporal resolution in the millisecond range. One of the most important applications of nonlinear EEG analysis itself is in epilepsy [2,3] because of the dynamic nature of the disease [4]. Furthermore, the underlying dynamics of epileptic EEG are highly nonlinear when compared to normal background EEG activity [5].

Nonlinear dynamical systems (also known as complex systems) have two main properties – determinism and recurrence [6]. A deterministic dynamical system can be defined as a system whose future behavior can be accurately predicted, given sufficient knowledge for the current state of the system exists. Let the current state of the system be given as z_n, such that $z_n \in \mathcal{M} \subseteq \mathbb{R}^m$, where $\mathcal{M}$ is an m-dimensional phase space attractor [7]. If there is an evolution operator $\Phi : \mathcal{M} \times \mathcal{Z} \mapsto \mathcal{M}$ such that $\Phi(z_n, t) = z_{n+t}$, then the system described by $(\mathcal{M}, \Phi)$ is said to be deterministic if the evolution operator Φ can precisely predict the state z_{n+t}, using the information present in z_n [7]. Recurrence is another property which can be used to characterize the nonlinear dynamics of a system [6]. Recurrence plot (RP) is a method for visualizing recurrences and was originally introduced by Eckmann et al. [8]. An RP is a two-dimensional graphical representation of a matrix (known as recurrence matrix – binary, square, and symmetric) that has an entry of one for times when two states are neighbors (as defined by some proximity criterion) in phase space and zero elsewhere [9]. A simple visual analysis of RPs can give an insight into the dynamics of the system. For example, the RP of a system exhibiting periodic dynamics contains long and non-interrupted diagonals, while for chaotic dynamics, the diagonals are much shorter [6]. On the other hand, for a stochastic system, the RP looks erratic and filled with many isolated black dots [10]. Apart from visual analysis, one can also derive quantitative measures for RPs using recurrence quantification analysis (RQA) [11,12] to investigate the dynamical properties of the system. For an excellent and detailed review on RPs, the reader is referred to [6]. Since the information from real world systems is usually in the form of a time series, one has to reconstruct the phase space using suitable methods like time-delay embedding [13] before applying RP based approaches. Note that, apart from RP-based methods, a host of other nonlinear methods have been introduced for time series analysis. Some of

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the most important and popular techniques are correlation dimension [14], Lyapunov exponent [15] and entropy-based measures [16]. An attractive feature of an RP based approach compared to other nonlinear approaches is that, it can be applied to short and non-stationary data [9].

In the last two decades, complex network theory has emerged as a popular tool to analyze complex and spatially extended systems [6]. It has found applications in melange of fields ranging from sociology to biological sciences [17]. Using network measures (local and global) [18,19], one can characterize the network structure and function of a complex system that is composed of many interacting elements [19]. Mathematically, a complex network can be represented by a graph $G = (\mathcal{N}, \mathcal{L})$, where the set $\mathcal{N} \equiv \{n_1, n_2, \ldots, n_N\}$ is known as vertices or nodes and the set $\mathcal{L} \equiv \{l_1, l_2, \ldots, l_K\}$ are the edges or links between those nodes [18]. For simplicity, we consider undirected graphs only. By integrating complex network theory with the concept of recurrence from dynamical systems theory, a new field of network-based time series analysis has been introduced that deals with the topological characterization of the time series using complex networks [9, 20]. Proximity networks are based on the concept of recurrences. Connectivity in such networks is defined in a data adaptive local manner [21]. Under proximity networks, a class of networks known as recurrence networks, which include k-nearest-neighbor networks, adaptive nearest neighbor (ANN) networks [22,23], and epsilon-recurrence networks (ε-networks) [9,20], reinterpret the binary recurrence matrix as an adjacency matrix of the complex network [6]. Specifically, an attractor's neighborhood is defined in terms of either fixed number of edges (k-nearest networks or ANN networks) [8,22–24] or fixed phase space volume (ε-networks) [9]. Such networks are also known as fixed mass and fixed volume networks respectively.

By quantifying the topology of the recurrence network using local and global measures from graph theory [25,26], the dynamical properties of the underlying complex system can be characterized [20]. Using global graph theoretic measures like the global clustering coefficient (C) and the average path length (L) for $G(N, K)$, Zou et al. [27] studied the identification of complex periodic windows in the Rössler system using ε-networks. It was found that for continuous-time dynamical system, C and L are in general greater for periodic dynamics compared to chaotic dynamics. Specifically, L is much smaller for a system exhibiting chaotic dynamics compared to periodic dynamics. In another study by Shimada et al. [24], using fixed mass networks (k-nearest neighbor networks) it was shown that chaotic dynamics can be characterized by small world networks (high C and small L). Xiang et al. [28] studied fixed mass networks (ANN networks) and found that L scales linearly with the network size for low-order periodic dynamics, but exponentially for chaotic dynamics. Also, the value of C is generally higher for periodic dynamics compared to chaotic dynamics.

Investigating the ability of network measures like C and L to characterize the dynamics of a system in the presence of observational noise is an important research question, as the real world data is seldom noise free. Thiel et al. [29] studied the influence of observational noise on RQA measures and found that these measures are susceptible to noise level of 20% or more (noise level is given as the standard deviation of the underlying noise-free process) and they proposed a threshold ε that is five times the standard deviation of the noise [29]. However, it has not been sufficiently studied yet, how the addition of observational noise can cause a change in the measures of recurrence networks like C and L for various threshold parameter (for example, various phase space volumes in case of ε-networks or number of edges in case of ANN networks). Also a study involving surrogate analysis method to test for the structural complexity of the data in the presence of noise by network measures like C or L has hitherto not been addressed. In general, a systematic study investigating the effect of observational noise on recurrence network measures and the ability of such measures to characterize the dynamical systems under increasing levels of noise is missing.

As mentioned before, nonlinear analysis is an important technique to understand the dynamical properties of the brain, especially in disorders like epilepsy. Andrzejak et al. [1] applied nonlinear measures like nonlinear prediction error P and effective correlation dimension $D_{2,eff}$ to different classes of EEG data: healthy EEG with eyes open and closed, EEG recordings between the seizures which is known as interictal EEG and EEG recordings of epileptic seizures which is known as ictal EEG. They reported strongest indication of nonlinearity for ictal EEG, while healthy EEG (eyes open) was compatible with quasilinear process. Gautama et al. [30] applied the method of delayed vector variance (DVV) for the data described in [1] and found indications of nonlinearity for both intracranial and surface EEG recordings. Given the potential of recurrence networks in characterization of dynamical systems and its reported advantages over other traditional non-linear measures in terms of its applicability to short and non-stationary data, its application in investigating the dynamical properties of EEG signals has not been fully explored. We have previously used the dataset described in [1] and characterized the underlying dynamics using fixed mass recurrence networks [31]. We found that the networks associated with ictal EEG are regular with high C and L, while the networks associated with interictal and surface EEG signals show small world property. However, we did not consider the effect of varying the threshold on the derived network measures. Also, we did not perform any surrogate analysis as reported in [1] to analyze structural properties of different classes of EEG signals.

Surrogate testing is an important tool in signal analysis [32]. In general, to use surrogate techniques for detecting nonlinearity, a null hypothesis is defined, which assumes that the original signal is compatible with a linear, stochastic, and stationary process, which is observed through a possibly nonlinear measurement function [1]. Based on this null hypothesis, a large number of signals (known as surrogates) are generated from the original signal such that the surrogates have the same linear autocorrelations as the original signal, but are otherwise random [32]. We then calculate a discriminating statistic for both the original signal and the surrogates. If the discriminating statistic of the original signal deviate from the surrogates, the null hypothesis is rejected. Rejection of the null hypothesis implies that the original signal is not consistent with the assumption of a linear, stochastic and stationary process and could indicate the presence of a possible nonlinear structure with a certain confidence level. Since recurrence network measures (like C and L) describe the structural properties of the attractors underlying a time series [33], these measures can be used as a discriminatory statistic to test for the structural complexity of the original data in conjunction with surrogates.

In this letter our aim is to investigate the ability of global network measures like C and L derived from recurrence networks to characterize dynamical systems under increasing levels of noise using simulated data and then to apply this method to study the structural properties of experimental signals like the EEG data. We organize our study to answer two specific questions

1. At what noise level is the structural complexity and detection of dynamical transitions as measured by recurrence network measures obscured?
2. Can recurrence network measures be used to analyze the different structural properties of healthy and epileptic EEG signals?

To answer question 1, we simulate the Rössler system to display periodic and chaotic dynamics, to which we systematically add in-