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Sloshing waves in a heated viscoelastic fluid layer in an excited rectangular tank

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ABSTRACT

In this paper, we have investigated the motion of a heated viscoelastic fluid layer in a rectangular tank that is subjected to a horizontal periodic oscillation. The mathematical model of the current problem is communicated with the linearized Navier–Stokes equation of the viscoelastic fluid and heat equation together with the boundary conditions that are solved by means of Laplace transform. Time domain solutions are consequently computed by using Durbin's numerical inverse Laplace transform scheme. Various numerical results are provided and thereby illustrated graphically to show the effects of the physical parameters on the free-surface elevation time histories and heat distribution. The numerical applications revealed that increasing the Reynolds number as well as the relaxation time parameter leads to a wider range of variation of the free-surface elevation, especially for the short time history.

1. Introduction

Usually, fluid in a container is influenced by the motion of the container. Such fluid assumes the form of waves, a phenomenon that is referred as sloshing. Liquid sloshing is such a complicated phenomenon. Depending on the type of disturbance and container shape, the free liquid surface can experience different types of motion including simple planar, non-planer, rotational, irregular beating, symmetric, asymmetric, quasi-periodic and chaos [1]. The sloshing of fluid in a tank is a well-studied problem that has applications in a number of practical situations. As indicated by Virella et al. [2], the problem is relevant to the safety of transporting fluids in tankers; Hermann and Timokha [3] also stress its relevance to the automotive, aerospace and shipbuilding industries. Fluid sloshing in road tankers may result in overturning of the vehicle, and resonant movement of fluid within ship cargo tanks is also of concern. These practical considerations are discussed further in the review article by Ibrahim and Pillipchuk [1]. They are reinforced by Frandsen [4], who also discusses the use of fluid-filled tanks to act as dampers on the motion of city buildings in high winds.

If sloshing is prolonged or accentuated, the wall of the container is damaged by the body force of the fluid, which in turn, results in unstable behavior of the whole mechanical system. Therefore, we need to clearly understand the sloshing-behavior characteristics of fluids. Published research into the sloshing phenomenon of general fluids theoretically analyzes the characteristics of hydrodynamic behavior through numerical analysis. It also addresses the design of apparatus and fluid–structure interaction, towards the reduction of sloshing. Liu and Lin, and Ikeda and Ibrahim analyzed sloshing using numerical analysis [5,6]. Abbas and Mansour investigated the damping of sloshing that arises from the use of baffles in fluid containers [7]. Also, Kim, Lee and Cho investigated optimization design technique for sloshing and free surface tracking for nonlinear liquid sloshing [8,9]. However, sloshing of general fluid is hard to control by itself; for reducing sloshing, additional structural elements are required in the fluid container [10–12].

Surface waves are primarily dominated by gravity and in the absence of solid boundaries viscosity usually has very little effect on the flow over a short period in time or over a short distance in space [13]. In other words, the effects of viscosity may become important only after many wave periods or after many wavelengths. It is quite common, therefore, that free surface flows of inviscid liquids are analyzed by the velocity potential theory. The problem becomes somewhat different when a wave encounters a body in its path, because of the sheared flow created by the body surface. But even in that case it is usually a common practice to deal with the free-surface effects and viscous effects separately. A typical example is linear wave interaction with an offshore structure. The interaction between the wave and the body can be analyzed by either wave-diffraction theory without viscosity or by viscous-flow theory without the free-surface effect, depending on the ratios of the characteristic dimension of the body to the wavelength and the wave amplitude [14]. Cases, however, do arise where the combined

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effect of the free surface and viscosity is important, some of which have been highlighted by Yeung and Yu [15]. A significant case is the flow near the liquid line of a floating body, i.e., at the intersection of the body surface and the free surface. Another example is the free surface flow of highly viscous fluid. In studying the behavior of the free surface interacting with the viscosity within a bounded region, it is assumed initially, common in potential flow analysis, that disturbance of the fluid is small and the flow is governed by the linearized Navier–Stokes (NS) equations. The justification and limitations of such an approximation have been discussed by Mei [16]. The analysis there is, however, based on the framework of boundary-layer theory. Such a case is easier than the case where the viscous effect is taken into account in the entire fluid domain. One difficulty in combined free-surface and viscous analysis lies at the intersection of the body surface and the free surface. Take a fixed body as an example: the no-slip condition in the NS equations suggests that the fluid particle there should remain stationary, but it can be observed experimentally that fluid moves up and down along the body surface. This difficulty was in fact mentioned by Lamb [17], and resolving it requires extensive experimental study such as that undertaken in [18]. Our current understanding of the flow structure near the intersection is still limited and many methods used to deal with the intersection are not entirely satisfactory. A commonly used scheme for water wave/structure interaction is based on two steps: (1) the NS equations are solved with the no-slip condition imposed on the body surface and (2) the motion of the intersection point is tracked through interpolation from the points on the free surface and near the intersection. This procedure has some clear defects. To ensure the result from the interpolation is accurate enough, the points used must be as close as possible to the intersection. However, if these points are sufficiently close to the body surface (for example when an extremely fine mesh is used in this region), the result will be the same as that based on the no-slip condition. Because of this difficulty, the analysis in this work remains modest. The no-slip condition on the body is replaced by a no-shear-force condition. The intention here is to show how the free surface will interact with the viscous flow, and results based on this model would be useful for this purpose. Indeed it was argued in [19] that the condition on the side walls may have little effect on the wave when analyzing Faraday's instability. Furthermore, the equivalent of a zero shear force condition on the side walls was also used by Loh and Rasmussen [20], who solved the full Navier–Stokes equations for this problem based on the finite-difference method.

When the temperature at the interface between two immiscible fluids is not uniform, a flow may be induced due to the temperature dependence of surface tension. This flow is usually called thermocapillary convection or Marangoni convection and can arise in a liquid–gas or a liquid–liquid interface subjected to a temperature gradient. As a consequence, the dependence of the surface tension upon the local temperature will create a shear stress on the liquid surface, which by viscous traction results in a Marangoni convection in the bulk of the liquid. In recent years, thermocapillary convection in systems with free boundaries have attracted much attention, mainly owing to their relevance in many processes of technological interest. Experimental study of thermocapillary convection is complicated by the presence of strong gravity convection under earthbound conditions and by residual mass–force accelerations and vibrations under microgravity conditions on board rocket probes and spacecraft. The simplest convective flow appears when a free surface of a single extended liquid layer is subject to a horizontal temperature gradient. As soon as a lateral heating is applied a basic flow settles down in the liquid layer and the resulting temperature profile is highly nonlinear. This flow destabilizes through an oscillatory instability that induces wave motions in simple fluids called hydrothermal waves (Smith and Davis [21]). Smith and Davis [21] showed that the instability mechanism even in this simple case is quite complex, resulting from the interplay between the basic flow and thermal or velocity disturbances. At low Prandtl numbers (Pr), hydrothermal waves propagate in a direction perpendicular to the horizontal temperature gradient, while at high Pr, they advance parallel to the temperature gradient. At intermediate Pr, the waves form an angle with the streamwise direction (Smith [22]). Later on, some authors (Parmentier et al. [23], Mercier and Normand [24]) extended these calculations by taking into account buoyancy effects and thermal transfer properties at the interface. Daviaud and Vince [25] reported the first observation of hydrothermal waves in a shallow layer of 0.65 cSt silicon oil. These observations were complemented by other authors considering different liquids and geometries (Mukolobwiez et al. [26] and Pelacho et al. [27]).

Although there is a numerous number of works has dealt with the study of the liquid sloshing, there remains considerable points are not still understood. This is mainly because sloshing is driven by free surface motion and related to many problems such as fluid–ship interaction, bubble formation, and probabilistic aspects. However, the reader may be referred to a rich series of papers on resonances and liquid sloshing, including the two and three dimensions, by Bridges et al. (see, e.g., Refs. [28–31]). In these works, they have discussed the free oscillation in a nearly square container for the case of standing waves. Also, they are interested in sloshing in shallow water in vessels that are undergoing a general rigid body motion.

The motivation of the present work is to explore analytically the non-Newtonian influence together with the heat transfer on the surface wave profile formatted at the free interface of a finite viscoelastic liquid layer within a horizontally oscillated rectangular container. From this point of view, the current paper represents an extension to some previous papers such as [32], that deals with the effect of the viscosity on the sloshing of the free surface of a Newtonian isothermal liquid layer [33], where the problem concerning with the nonlinear sloshing dynamics of an isothermal ideal liquid layer within a horizontally oscillating rectangular tank, and Herczyński and Weidman [34] who have presented an analytical and experimental study on the periodic oscillation of free containers driven by an inviscid liquid sloshing where the rectilinear horizontal motion of the containers is assumed to be frictionless.

The problem is formulated mathematically in Section 2. The basic state solution of the problem is obtained in Section 3. We then proceed to obtain the solution of the perturbed linearized problem by means of Laplace transforms. Some important limiting cases are investigated at the end of this section. In Section 4, we have presented the procedure of the numerical approach utilized to assign the perturbed quantities in terms of the time parameter and then some numerical computations are presented to demonstrate the effects of the various parameters on the behavior of the system. The concluding remarks are summarized in Section 5.

2. Problem formulation

We consider the problem of viscoelastic nature fluid flow described by the Maxwell constitutive relation in a rectangular tank of length l, which undergoes a horizontal oscillation motion. A cartesian coordinate system O–xz is defined so that its origin is located at the center of the mean free surface and z-axis points upwards. The vertical sidewalls are insulated and the base of the tank is maintained at constant temperature T_b while, the fluid free surface is subjected to Newtonian cooling with external ambient temperature T_a and convective heat transfer coefficient h_g. In addition, we will suppose that interfacial tension varies linearly with temperature, which can be expressed as follows [35]

$$\sigma = \sigma_0 - \Gamma(T - T_a), \tag{1}$$