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Influence of electrostatic field on the Weiss oscillations in graphene

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ABSTRACT

Recent theoretical works have predicated the appearance of Weiss oscillations in the magnetoconductivity with a one-dimensional periodic electrical or magnetic modulation in graphene. This paper further explores the electrostatic field effect on the Weiss oscillations in the presence of crossed uniform in-plane electric field and perpendicular magnetic field that is weakly and periodically modulated along one direction. We find that the oscillation amplitude (OA) of Weiss oscillations and the value of conductivity are both shown to increase as the electric field E increases for a given magnetic field B. More interestingly, the electric field leads to an abrupt disappearance of the Weiss oscillations, when the value of electric to magnetic field ratio approaches a threshold value, i.e., $\gamma_e = \frac{E}{\upsilon_F B} = 1$. These phenomena, not known in the conventional 2D electron gas, are a consequence of the anomalous spectrum of electron in graphene.

1. Introduction

Owing to the progress in experimental methods, graphene (or a graphite monolayer) is now attracting increasing interest in the field of the physics of electronic systems with reduced dimensionality [1–4]. It is promising for application in nanoelectronics because of the exotic chiral features [5–9] in its electronic structure. To date, graphene has led to some of the most startling discoveries in condensed matter physics in recent years. In particular, these anomalous phenomena are found to be tied to the remarkable 'relativistic-like' spectrum of electrons and holes in graphene, which makes graphene important and interesting in physics. One of those that have been experimentally testified is the abnormality of the 2D quantum Hall effect [10–12]. Earlier, it was found, in the conventional 2DEG, that the artificially created periodic potentials in the submicrometer range led to the appearance of Weiss oscillations in the magnetoresistance [13–15]. This type of oscillation was shown to be periodic in the reciprocal magnetic field ($\sim 1/B$) like the Shubnikov–de Haas (SdH) oscillations, which appear due to the interplay of the quantum Landau levels with the Fermi energy, and serve as a powerful technique to investigate the Fermi surface and the spectrum of electron excitations. However, it should be stressed here that such two types of oscillations present a different relation between the period and the electron density (n_e), i.e., the period for Weiss oscillations varies with $\sqrt{n_e}$, but with n_e for SdH oscillations. Essentially, the Weiss oscillations are a consequence of the commensurability of the electron cyclotron orbit diameter at the Fermi energy and the period of the above electrical modulation. Recently, Matulis and Peeters have further investigated the electrical transport of Dirac electrons in graphene with the electrical modulation [16]. Along the same lines, Tahir and Sabeeh have studied low temperature magnetotransport of electrons in graphene subjected to the magnetic modulation [17]. However, these researches do not involve the electrostatic field effects on the magnetoresistance in graphene.

To date, many investigations have focused their attention on the electronic and magnetic properties of graphene and graphene nanoribbons (GNRs) [18–31]. Specifically, Lukose et al. [18] have firstly found the Landau spectrum in graphene gets scaled by an electric field dependent dimensionless parameter ($\gamma_e = E/B\upsilon_F$), i.e., as γ_e increases, the Landau levels (LLs) spacings decrease and finally the Landau spectrum collapses at threshold value $\gamma_e = 1$. Thereafter, Peres and Castro [19] have obtained the full algebraic solution of the mathematical problem. Furthermore, the authors in [20–22] have explored the electromagnetic properties of GNRs, revealing spectacular effects arising from the confinement of electron and hole gases. One of the most remarkable findings is that electrons and holes can be specularly reflected, such as billiard particles, at the edges of a GNR [23]. Besides, some recent works

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report on the individual and combined effects of an electric and magnetic field on the ballistic transport and optical conductance of GNRs [24,25]. Zhang and Ma et al. [26,27] have predicted the modulation of the de Haas–van Alphen (dHvA) effect in graphene and GNRs by electric field, which has drawn much attention in both theoretical and experimental groups [28–31]. Especially, Reis et al. [28–30] have further studied the electrocaloric and magnetocaloric effects on graphenes and GNRs in the presence of external fields, which is a quite meaningful case. Moreover, the magnetocaloric effect has been also explored with a periodic and harmonic magnetic field in [32].

Motivated by this, we calculate the diffusive conductivity in the presence of crossed uniform in-plane electric field and perpendicular magnetic field that is weakly and periodically modulated along one direction. Furthermore, we compare our result with that without electric field to highlight the electric field effect on low temperature magnetotransport of Dirac electrons in graphene. Finally, we find the Weiss oscillations in the magnetoconductivity are more robust with respect to the electric field. Specifically, the oscillation amplitude (OA) of Weiss oscillations and the value of conductivity both increase as E increases with a given B, but the opposite behavior for the Shubnikov–de Haas (SdH) oscillations is seen. More interestingly, the Weiss and the SdH oscillations both abruptly vanish at $\gamma_e = 1$, which is attributed to the anomalous electric field effect on the spectrum of graphene.

This paper is organized as follows. In Section 2, a brief introduction is given to the 2D model for graphene, and the precise energy eigenvalues and eigenstates in the presence of crossed uniform electric and magnetic fields are obtained analytically. In Section 3, the electric field effects on the electrical conductivity with periodic magnetic modulation in graphene are described, and the results compared with that without electric field. Furthermore, the asymptotic expression is derived for the electric field dependent magnetoconductivity. In the last section, we present brief summary and conclusions.

2. Energy spectrum

In present work, we directly start from the massless Dirac equation $\mathbf{p}_\mu \boldsymbol{\gamma}^\mu_{ik}\Psi_k = 0$ in the lowest order approximation to calculate the energy eigenvalues and eigenstates for graphene electrons in the presence of crossed uniform electric and magnetic fields. Our method is strikingly different from that employed by the authors in [18], who have solved the problem by transforming the original system into a case with a null electric field, in terms of a Lorentz boost transformation. But we should point out that the eigenstates for graphene electrons in [18] are not the integrated form with the electric field. However, the precise eigenstates are crucial for the calculation of the diffusive conductivity and thus we should firstly solve this problem here. In crossed electric [$\mathbf{E} = (-E, 0, 0), U = Eex$] and magnetic [$\mathbf{B} = (0, 0, B), \mathbf{A} = (0, Bx, 0)$] fields, the low energy excitations are described by the 2D Dirac-type Hamiltonian [18,26,27],

$$H_0 = \upsilon_F \boldsymbol{\alpha} \cdot (-i\hbar\nabla + e\mathbf{A}) + \mathbf{I}eEx, \tag{1}$$

in which the $\boldsymbol{\alpha}$ are the Pauli matrices and υ_F characterizes the Fermi velocity (i.e. $\upsilon_F = 1.0 \times 10^6$ m/s). Following the Landau and Lifshitz [33], we have

$$\left\{-2m\upsilon_F^2\left[\frac{p_x^2}{2m} + \frac{m\omega_c^2}{2}(x - x_c)^2\right] + \frac{(B\epsilon - p_y E)^2}{B^2(1 - \gamma_e^2)} + e\hbar B\upsilon_F^2\boldsymbol{\alpha}_z + ie\hbar\upsilon_F E\boldsymbol{\alpha}_x\right\}\Psi = 0, \tag{2}$$

in which we define the cyclotron frequency as $\omega_c = eB(1 - \gamma_e^2)^{1/2}/m$ with $\gamma_e = E/B\upsilon_F$, and the centers of the x-dependent orbitals as $x_c = [\ell_c^2 k_y - \epsilon\gamma_e^2/(eE)]/(1 - \gamma_e^2)$ with the magnetic length $\ell_c = \sqrt{\hbar/eB}$. The symbol ϵ denotes the Dirac equation eigenvalue. The solutions of Ψ are hence the product of orbital $[\phi_n(x, y)]$ and spinor $[\chi_\pm(E, B)]$ functions, since the first two terms depend only on orbital coordinates, and the last two on spinor coordinates. Since the present coordinate system is chosen with $\mathbf{E}$ along the x direction, the orbital eigenfunctions may be taken as plane waves in the y direction, and thus we seek the solutions in the separable form $\Psi \sim \exp(ik_y y)\phi_n(x)\chi_\pm(E, B)$. Substitution of such Ψ into Eq. (2) inevitably leads to the two equations, respectively, for $\phi_n(x)$

$$\left\{-2m\upsilon_F^2\left[\frac{p_x^2}{2m} + \frac{m\omega_c^2}{2}(x - x_c)^2\right] + \frac{(B\epsilon - p_y E)^2}{B^2(1 - \gamma_e^2)}\right\}\phi_n(x) \equiv \eta_n\phi_n(x) \tag{3}$$

and for $\chi_\pm(E, B)$

$$\left(e\hbar B\upsilon_F^2\boldsymbol{\alpha}_z + ie\hbar\upsilon_F E\boldsymbol{\alpha}_x\right)\chi_\pm(E, B) \equiv \eta_\pm\chi_\pm(E, B). \tag{4}$$

The first term of Eq. (3) is the familiar one-dimensional harmonic oscillator Hamiltonian, so the eigenvalues and eigenfunctions are known. Solving Eq. (4), we can get the eigenvalues

$$\eta_\pm(E, B) = \pm e\hbar B\upsilon_F^2\sqrt{1 - \gamma_e^2}, \tag{5}$$

and the eigenspinors

$$\chi_+^T = (\gamma_e\varpi_m,\ 0,\ 0,\ i\varpi_m\varpi_n),$$
$$\chi_-^T = (0, \gamma_e\varpi_m,\ -i\varpi_m\varpi_n,\ 0) \tag{6}$$

with

$$\varpi_m = \left(\gamma_e^2 + \varpi_n^2\right)^{-1/2},$$
$$\varpi_n = 1 - \left(1 - \gamma_e^2\right)^{1/2}. \tag{7}$$

Finally, the precise Ψ could be obtained in the following form

$$\Psi_{n,k_y} = \frac{\exp(ik_y y)}{\sqrt{2L_y\ell_c^*}}\begin{pmatrix} -i\Phi_{n-1}(\xi) \\ \Phi_n(\xi)\end{pmatrix}\chi_\pm(E, B), \tag{8}$$

in which

$$\Phi_n(\xi) = \frac{e^{-\xi^2/2}}{\sqrt{2^n n!\sqrt{\pi}}}H_n(\xi) \tag{9}$$

is expressed in the normalized Hermitian polynomials. We express $\xi = (x + x_c)/\ell_c^*$ with $\ell_c^* = l_c/(1 - \gamma_e^2)^{1/4}$ and $x_c = \ell_c^2 k_y - \mathrm{sgn}(n)\sqrt{2|n|}\ell_c^*\gamma_e$, which can be compared with the conventional expression $x_c^{con} = \ell_c^2 k_y - \upsilon_F^2\gamma_e^2 m/eE$. The imposition of periodic boundary conditions $\Psi(x, y + L_y) = \Psi(x, y)$ over some suitably large length L_y leads to the allowed values $k_y = 2\pi\ell/L_y$ ($\ell = 0, \pm1, \pm2, \ldots$). The localized character of the bound-state eigenfunctions $\phi_n(x)$ ensures that the electronic motion is bounded in the x direction, so that no runaway electrons develop under these conditions. Finally, the equation $(\eta_n + \eta_\pm)\Psi = 0$ provides the eigenvalue

$$\epsilon_{n,k_y} = \mathrm{sgn}(n)\omega_g\sqrt{|n|} + \hbar\upsilon_F\gamma_e k_y, \tag{10}$$

where the integer n denotes the LLs, and k_y is a good quantum number corresponding to the translation symmetry along the y axis. The symbol ω_g is given by $\omega_g = \sqrt{2}\hbar\upsilon_F(1 - \gamma_e^2)^{3/4}/\ell_c$ with $|\gamma_e| < 1$ yielding to the demand of Lorentz covariant. Eq. (10) reveals that the electric field could modulate the LLs spacing and eventually cause a collapse at $\gamma_e = 1$, as demonstrated in [18,19,26,27]. From another perspective, according to the Bohr–Sommerfeld quantization condition $\int_{x_1}^{x_2}\sqrt{(\varepsilon - eEx)^2/\upsilon_F^2 - (p_y - eBx)^2}dx =$