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Flat acoustic lens by acoustic grating with curled slits

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A B S T R A C T

We design a flat sub-wavelength lens that can focus acoustic wave. We analytically study the transmission through an acoustic grating with curled slits, which can serve as a material with tunable impedance and refractive index for acoustic waves. The effective parameters rely on the geometry of the slits and are independent of frequency. A flat acoustic focusing lens by such acoustic grating with gradient effective refractive index is designed. The focusing effect is clearly observed in simulations and well predicted by the theory. We demonstrate that despite the large impedance mismatch between the acoustic lens and the matrix, the intensity at the focal point is still high due to Fabry–Perot resonance.

1. Introduction

Over the past decade, propagation of acoustic waves through rigid plates perforated with periodical subwavelength apertures, known as acoustic gratings, has attracted a lot of interest [1–19]. The existence of Fabry–Perot (FP) resonance in the apertures (slits or holes) or the coherent diffractions effect has led to a variety of potential applications, including broadband transmission enhancement [1–6], acoustic collimation [7], subwavelength imaging [8–10] and acoustic focusing [15,16,19]. Lu et al. reported the phenomenon of extraordinary acoustic transmission, observed at low frequencies, through a one-dimensional acoustic grating [1]. The unusual phenomena aroused great interest in the effective elastic properties of acoustic gratings. Cai et al. derived the expressions for transmission and reflection coefficients and studied the effective parameters of acoustic gratings [5]. They showed that an acoustic grating with straight subwavelength apertures could serve as a homogenous medium with high refractive index, and an acoustic wave-focusing lens with curved surfaces was designed accordingly. Later, the acoustic grating was considered as an effective slab with anisotropic mass density [9–11], which better describes the properties of the acoustic grating and explains well the case of oblique incidence. Therefore, by using the calculated effective mass density, an anisotropic super-lens, which could overcome the diffraction limit, was designed based on the acoustic grating [9,10]. Although many acoustic devices were developed, most of them, especially those relying on the FP resonance, had the limitation of large thickness, leading to bulky acoustic devices at low frequencies.

Kock and Harvey discovered the path length delay-type lenses to tailor the effective refractive index for sound waves in 1949 [20]. Very recently, Liang and Li presented a novel acoustic metamaterial, named as coiling-up space [12], by using a similar approach. This kind of acoustic metamaterial, which was composed of curled slits, could induce unusual acoustic properties, such as negative effective mass density, at low frequencies without any locally resonating units. Li et al have further stated that this acoustic metamaterial could be used to overcome the size limitation for the acoustic devices [14], and they have also successfully achieved an ultrathin acoustic lens by coiling up space [15]. However, the effective parameters of the acoustic lens were only numerically studied, and the acoustic lens was designed with a row of separate units of coiling structure. This, in turn, made the underlying mechanism of the acoustic lens more complicated. For example, acoustic waves could simultaneously pass through the units and the interspace between the units.

In this work, we use the coupled-mode method [3,4] to derive the expressions for transmission and reflection coefficients of an acoustic grating with curled slits. By comparing the two coefficients with those obtained for acoustic waves passing through a homogeneous slab, we have obtained the effective impedance and refractive index of the acoustic grating. Different from previous works, in which the effective parameters are dispersive [14, 15], we show that the effective parameters of the acoustic grating do not depend on frequency. Based on the effective medium parameters, we design a flat subwavelength acoustic lens using a gradient acoustic grating. The numerical results show that the flat

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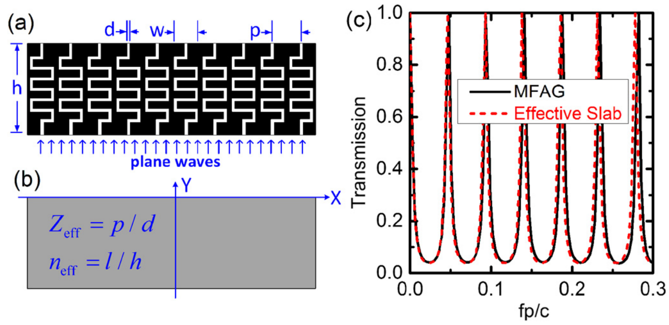

Fig. 1. (Colour on-line.) (a) Sketch of the MFAG, which can serve as (b) a homogenous slab with effective impedance Z_{eff} and diffractive index n_{eff}. The geometry parameters used in (a) are $d = 0.1p$, $h = 6p$, $w = 0.8p$, and $m = 6$. (c) The black solid line and red dashed line depict the transmission spectrum of the MFAG and the effective homogenous slab, respectively, where the plane acoustic waves are normal incident from bottom. Here frequency f is normalized by c/p.

lens can focus acoustic waves and the focal point is well predicted by the effective medium.

2. Effective medium description

The structure of the acoustic grating is shown in Fig. 1(a), in which a steel slab with periodic curled slits is placed in air. We name such a structure a Multiple Folded Acoustic Grating (MFAG). The thickness of the plate, width of the slits and the length of one horizontal segment of the MFAG are h, d and w, respectively. If a slit is folded up m times, the total length of the slit is $l = h + mw$. The distance between the adjacent slits is p, which is also referred to as periodicity. h, d, w and p are relatively small compared to the wavelength in our study. Thus, we may use a homogenous slab with anisotropic mass density [10] to model such a MFAG. The effective mass density along the x-direction is suggested to be infinite, whereas the effective mass density along the y-direction can be obtained by comparing the normal transmission and reflection coefficients of the MFAG with those of a homogenous slab. To derive the transmission coefficient of the MFAG, we employ the coupled-mode method, which was firstly introduced in the study of enhanced optical transmission [3,21] and later further developed into enhanced acoustic transmission [1,4]. In the method, the steel plate is regarded rigid, because of the huge mismatch in both velocity and mass density between the steel and the air. P_1 and P_3 refer to the pressure field below and above the slab. They can be expressed as:

$$P_1(x, y \le -h) = \sum_{j=-\infty}^{\infty} \left(\delta_{0j} e^{i\alpha_j (y+h)} + r_j e^{-i\alpha_j (y+h)}\right) e^{iG_j x}$$

$$P_3(x, y \ge 0) = \sum_{j=-\infty}^{\infty} t_j e^{i\alpha_j y} e^{iG_j x}, \tag{1}$$

where δ_{0j} is the Kronecker delta and j is an integer. Here r_j and t_j denote the normalized pressure field amplitudes of the jth reflected and transmitted waves, respectively. $G_j = k_x + 2j\pi/a$ is the parallel momentum along the plate surface of the jth diffraction order, with k_x being the x component of the incident wave vector $k = \omega/c$ (ω is the angular frequency and c is the sound velocity in air). $\alpha_j = \sqrt{k^2 - G_j^2}$ is the momentum along the y-direction. The velocity fields along the y-direction are then obtained by the relationship $\rho \partial v/\partial t = -\partial P/\partial y$, where ρ is the mass density of air.

Inside the slits (for example $|x| < d/2$), only the zeroth-order waveguide mode can propagate, because the width of the slit, d, is much smaller than the wavelength [22]. The pressured field of acoustic wave propagating inside the curled slit can be approximated by the field inside a straight waveguide with length l, i.e., $P_2^l(|x| \le d/2, -l \le y \le 0) = Ae^{ik(y+l)} + Be^{-iky}$. Therefore, the phase accumulation inside the curled slits is equal to kl and the pressure field inside the curled slit can be expressed as:

$$P_2\left(|x| \le d/2, -h \le y \le 0\right) = Ae^{ik'(y+h)} + Be^{-ik'y}, \tag{2}$$

where A and B are the corresponding amplitudes of the upward and downward propagating waves, respectively, and k' is the equivalent wave vector. Then the value of k' should satisfy the relation $k'h = kl$, which describes the exact phase change inside the curled slits. The velocity field inside the slits is obtained in a similar manner. By matching the boundary conditions at $y = -h$ and $y = 0$, where the pressure P and normal velocities v_y should be continuous, we can obtain the transmission coefficient. For simplicity, in the case of narrow slits (with $a \ll p$), low frequency (with $j = 0$) and normal incident (with $k_x = 0$) are assumed, the expression of the transmission coefficient along the y-direction is given as:

$$t_0 = \frac{4(p/d)e^{ik'h}}{[1 + (p/d)]^2 - [1 - (p/d)]^2 e^{2ik'h}}$$

$$r_0 = \frac{[1 - (p/d)^2] - [1 - (p/d)^2]e^{2ik'h}}{[1 + (p/d)]^2 - [1 - (p/d)]^2 e^{2ik'h}}, \tag{3}$$

On the other hand, the transmission and reflection amplitude of plane acoustic waves normally incident to a homogeneous slab with thickness h can be expressed as:

$$t_{\mathrm{slab}} = \frac{4Z_{\mathrm{eff}} e^{ikn_{\mathrm{eff}}h}}{(1 + Z_{\mathrm{eff}})^2 - (1 - Z_{\mathrm{eff}})^2 e^{2ikn_{\mathrm{eff}}h}}$$

$$r_{\mathrm{slab}} = \frac{(1 - Z_{\mathrm{eff}}^2) - (1 - Z_{\mathrm{eff}}^2)e^{2ikn_{\mathrm{eff}}h}}{(1 + Z_{\mathrm{eff}})^2 - (1 - Z_{\mathrm{eff}})^2 e^{2ikn_{\mathrm{eff}}h}}, \tag{4}$$

where Z_{eff} and n_{eff} are, respectively, the relative impedance ($Z_{\mathrm{eff}} = Z_{\mathrm{slab}}/Z_{\mathrm{air}}$) and refractive index ($n_{\mathrm{eff}} = n_{\mathrm{slab}}/n_{\mathrm{air}}$) of the slab. Comparing Eq. (3) with Eq. (4), we observe certain correspondence between the MFAG and the effective slab, when the effective parameters of the slab satisfy the relationship $Z_{\mathrm{eff}} = p/d$ and $n_{\mathrm{eff}} = l/h$. We would like to remind that the obtained effective parameters Z_{eff} and n_{eff} are along the y-direction.

To verify the analytical results, numerical simulations for transmission coefficients of acoustic waves normally incident onto the MFAG have been performed. We use COMSOL Multiphysics, a commercial package based on the finite-element method, to calculate the transmission spectrum. As shown in Fig. 1(c), the black solid curve denotes the simulated transmission spectrum of plane waves normally incident from the bottom of MFAG. The geometry parameters used are $d = 0.1p$, $h = 6p$, $w = 0.8p$, and $m = 6$. We also analyze the transmission spectrum for its effective counterpart, a homogenous slab, with effective impedance $Z_{\mathrm{eff}} = 10$ and refractive index $n_{\mathrm{eff}} = 1.8$ along the y-direction. The results are plotted in the red dashed curve in Fig. 1(c), which agree well with the black solid curve, suggesting the effective model is valid for the transmission coefficient.

3. Flat acoustic lens

To demonstrate an application of this MFAG, we design an acoustic lens using the obtained effective parameters. The acoustic lens has been widely studied [23–26] by using phononic crystals or acoustic metamaterials. Cervera et al. experimentally characterized a lens with curved surface by using 2D phononic crystals [23]. Hakansson et al. realized a flat acoustic lens by inverse design [24]. Torrent and Sanchez-Dehesa introduced the idea of gradient index lens by using a homogenization approach at low frequencies [25].