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Multiple routes transmitted epidemics on multiplex networks

Dawei Zhao [a,b,c], Lixiang Li [a,b], Haipeng Peng [a,b,*], Qun Luo [a,b], Yixian Yang [a,b]

[a] Information Security Center, State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, P.O. Box 145, Beijing 100876, China
[b] National Engineering Laboratory for Disaster Backup and Recovery, Beijing University of Posts and Telecommunications, Beijing 100876, China
[c] Shandong Provincial Key Laboratory of Computer Network, Shandong Computer Science Center, Jinan 250014, China

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ABSTRACT

This letter investigates the multiple routes transmitted epidemic process on multiplex networks. We propose detailed theoretical analysis that allows us to accurately calculate the epidemic threshold and outbreak size. It is found that the epidemic can spread across the multiplex network even if all the network layers are well below their respective epidemic thresholds. Strong positive degree–degree correlation of nodes in multiplex network could lead to a much lower epidemic threshold and a relatively smaller outbreak size. However, the average similarity of neighbors from different layers of nodes has no obvious effect on the epidemic threshold and outbreak size.

1. Introduction

In recent years, various types of epidemics have occurred frequently and spread around the world, causing not only great economic losses, but also widespread public alarms. For example, the intense outbreak of SARS caused 8098 reported cases and 774 deaths. Within weeks, SARS spread from Hong Kong to infect individuals in 37 countries in early 2003 [1]. An outbreak of mobile viruses occurred in China in 2010. The 'Zombie' virus attacked more than 1 million smart phones, and created a loss of $300,000 per day [2]. And we have also witnessed how social networks were used for citizens to share information and gain international support in the Arab Spring [3]. In view of these situations, it is thus urgent and essential to have a better understanding of epidemic process, and to design effective and efficient mechanisms for the restraint or acceleration of epidemic spreading.

Valid epidemic spreading models can be used to estimate the scale of an epidemic outbreak before it actually occurs in reality and evaluate new and/or improved countermeasures for the restraint or acceleration of epidemic spreading. In the last decade, there have been extensive studies on the modeling of epidemic dynamics [4–10], and various protection strategies have been proposed and evaluated [11–18]. However, these existing researches have been dominantly focusing on the cases that epidemics spread through only single transmission route. While in reality, many epidemics can spread through multiple transmission routes [19] simultaneously. For example, it has been well recognized that AIDS can propagate via three routes simultaneously including sexual activity, blood and breast milk; rumor or information can spread among human through verbal communication and social networks; malwares can move to computers by P2P file share, email, random-scanning and instant messenger [20]; and some mobile malwares can attack smart phones through both short messaging service (SMS) and bluetooth (BT) at the same time [21]. In this letter, the epidemic which spreads via single transmission route and multiple transmission routes are called single route transmitted epidemic and multiple routes transmitted epidemic, respectively. When a multiple routes transmitted epidemic is spreading on a network, the network node could be infected via one of the transmission routes even if it cannot be infected via the other routes. And the node can be infected with a higher probability if it can be infected via more than one transmission route of the epidemic. Therefore, the range and the intensity of the multiple routes transmitted epidemic will be greater than those of the traditional single route transmitted epidemic. Meanwhile, different transmission routes are supported by different networks. For instance, the underlying network of the mobile malware which propagates via SMS is an SMS network formed based on the social relationships among mobile users. And the BT network formed according to the geographically positions of mobile devices is the underlying network of the mobile malware which can spread through BT. Therefore, the underlying network of the multiple routes transmitted epidemic is actually a multiplex network [22–25], rather than a single network. Multiplex network can be regarded as a set of coupled layered networks

* Corresponding author. Tel.: +86 10 62282264.
E-mail address: penghaipeng@bupt.edu.cn (H. Peng).

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Table 1
Symbols used in this letter and their meanings.

Symbols	Meanings
ASN	Average similarity of neighbors from different layers of nodes.
DDC	Correlation of nodes' degrees in one layer and that in another layer.
λ_A	The probability that a susceptible node is infected only via route-A.
λ_B	The probability that a susceptible node is infected only via route-B.
λ_C	The probability that a susceptible node is infected via route-A and route-B simultaneously.
(λ_A, λ_B)	The spreading rate of a two routes transmitted epidemic, where λ_A and λ_B are the spreading rates of this epidemic when spreading on layer-A and layer-B, respectively.
k_A	Degree of node in layer-A.
k_B	Degree of node in layer-B.
k_C	The number of same neighbors of node in layer-A and layer-B.
k_M	Vector degree of node on multiplex network.
$\{(\lambda_A, \lambda_B)_c\}$	Epidemic threshold of multiplex network with two layers.
s	Outbreak size of epidemic.

in which each layer could have very particular features different from the rest and support different dynamical processes. Based on the above analyses, the study of multiple routes transmitted epidemic on multiplex network is definitely a very meaningful and necessary topic.

To the best of our knowledge, the theory describing the multiple routes transmitted epidemic process on multiplex network has not been fully developed yet. In this letter, we propose and evaluate a two routes transmitted epidemic spreading on multiplex network with two network layers following the typical Susceptible–Infected–Removed (SIR) model [6,7]. But the proposed research methods can be easily extended to analyze the epidemics which spread via any number of transmission routes. By mapping the SIR model into the bond percolation [7], we develop equations which allow accurate calculations of epidemic threshold [6] of the multiplex network and outbreak size [6] of the epidemic. It is found that the epidemic can spread across the multiplex network even if the two network layers are well below their respective epidemic thresholds. We also introduce two quantities for measuring the level of inter-similarity between these two layers. One is the average similarity of neighbors (ASN) from different layers of nodes. ASN evaluates how many neighbors of nodes in one layer are also their neighbors in another layer. We find that both epidemic threshold and outbreak size are not significantly affected by the ASN. The second quantity is the degree–degree correlation (DDC) of nodes which describes the correlation of nodes' degrees in one layer and that in another layer. Positive DDC indicates that high degree nodes in one layer are also high degree ones in another layer, and vise versa. It is found that strong positive DDC could lead to a clearly lower epidemic threshold and a relatively smaller outbreak size.

Some symbols used throughout this letter and their meanings are summarized in Table 1.

2. Models and analysis

2.1. Multiple routes transmitted epidemic spreading model

The epidemic spreading model adopted here is the Susceptible–Infected–Removed (SIR) model which is the most basic and well-studied epidemic spreading model [6,7]. In the SIR model, the nodes of the network can be divided into three compartments, including susceptibles (S, those who are prone to be infected), infectious (I, those who have been infected), and recovered (R, those who have recovered from the disease). At each time step, a susceptible node becomes infected with probability λ if it is directly connected to a infected node. The parameter λ is called the spreading rate. Meanwhile, an infected node becomes a recovered node with probability δ.

In this letter, what we study is a simple case that a two routes transmitted epidemic spreads among network individuals.

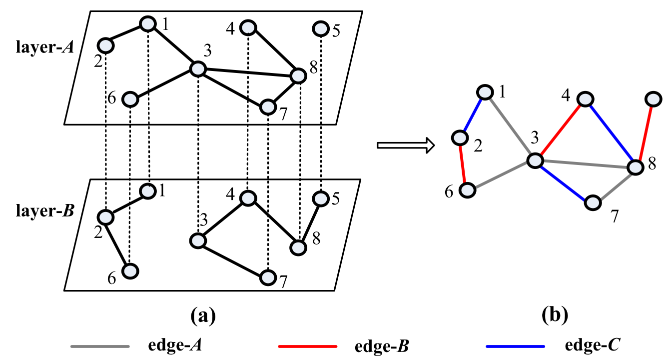

Fig. 1. (Color online.) (a) A multiplex network with two network layers, i.e., layer-A and layer-B. (b) The multiplex network in the form of the superposition of layer-A and layer-B.

Therefore, we need to specify the corresponding epidemic spreading processes separately. It is assumed that these two transmission routes of the epidemic are route-A and route-B, respectively. Then we assume that a susceptible node becomes infected with probability λ_A or λ_B if it can be infected only through route-A or route-B. Besides, if a susceptible node can be infected via route-A and route-B simultaneously, the probability that this susceptible node becomes infected is assumed to be λ_C. Obviously, $\lambda_C = 1 - (1 - \lambda_A)(1 - \lambda_B)$. Meanwhile, an infected node becomes a recovered node with probability δ. Without loss of generality, we let $\delta = 1$.

2.2. Multiplex networks model

Since different transmission routes are supported by different networks, the underlying network of a two routes transmitted epidemic should be a multiplex network with two network layers. In this section, as shown in Fig. 1(a), we propose a multiplex network model which contains two network layers, i.e., layer-A and layer-B. Nodes are the same in both layers, and layer-A and layer-B are the underlying networks of the epidemic spreading via route-A and route-B, respectively. Fig. 1(b) shows this multiplex network in the form of the superposition of layer-A and layer-B. In the rest of this paper, the multiplex network, unless otherwise noted, is assumed to be the network in the form of the superposition of layer-A and layer-B.

Each node in the proposed multiplex network has up to three types of edges where edge-A belongs only to layer-A, edge-B belongs only to layer-B, and edge-C belongs to both layer-A and layer-B. The vector degree $k_M \equiv (k_A - k_C, k_B - k_C, k_C)$ is used to characterize the node of multiplex network, where $k_A - k_C$, $k_B - k_C$ and k_C represent the numbers of edge-A, edge-B and edge-C of the node, respectively. The numerical value of vector degree of the