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Electrostatic instability in magnetically confined inhomogeneous plasma driven by nonlinear force

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ABSTRACT

A theoretical investigation on amplification of electrostatic ion acoustic wave in magnetically confined plasma has been presented in this paper. This investigation considers nonlinear wave–particle interaction process, called plasma maser effect, in presence of drift wave turbulence supported by magnetically confined inhomogeneous plasma. The role of associated nonlinear dissipative force in this effect in a confined plasma has been analyzed. The nonlinear force, which arises as a result of resonant interaction between electrons and modulated fields, is shown to drive the instability. Using the ion fluid equation and the ion equation of continuity, the nonlinear dispersion relation of a test ion acoustic wave has been derived, and the growth rate of ion acoustic wave in presence of low frequency drift wave turbulence has been estimated using Helimak data.

1. Introduction

The phenomena of enhanced high frequency fluctuations in the presence of low frequency turbulences are often observed in laboratory and space plasmas [1]. Already Auroral Kilometric Radiation (AKR) has been explained as an enhanced extraordinary mode (X-mode) radiation due to the presence of electrostatic ion cyclotron turbulence [2].

In magnetically confined plasmas, this process is a quite relevant one. Itoh et al. [3], in their theoretical investigation, have predicted excitation of Geodesic acoustic modes (GAM) by drift waves using a wave-kinetic approach in a toroidal plasma. In a subsequent theoretical investigation, Chakrabarti et al. [4] have predicted resonance excitation of GAM by a spectrum of drift waves in a toroidal plasma.

Anomalous transport in toroidal plasma is an important issue, and the cause of this anomalous transport has been ascribed to plasma turbulence driven by a family of drift waves [5]. Drift waves are low frequency electrostatic turbulences in confined plasma producing dominant mechanism for the transport of particles, energy and momentum across magnetic field lines [6]. One essential element of the nonlinear dynamics of drift turbulence is the emergence of sheared zonal flows (ZFs), which are linearly stable $\mathbf{E} \times \mathbf{B}$ drifts within the magnetic surface, driven by the nonlinear transfer of energy from the drift waves into the ZF [7].

The drift modes have closed phase relation with thermal particles, and because of that, there is a strong possibility that these modes take part in wave energy exchange mechanism through wave–particle interaction process.

In a toroidal geometry, ion acoustic wave can couple nonlinearly with drift wave [6]. Works in the Columbia Linear Machine (CLM) device also point to the possibility that drift wave could couple nonlinearly to ion acoustic wave, which appears to act as an energy sink to drift wave turbulence in these experiments [7].

Here in this paper, we intend to investigate amplification of ion acoustic wave in presence of drift wave turbulence in a magnetically confined geometry. Recently, effort [8] has been made to explain electrostatic instability in inhomogeneous plasma in presence of drift wave turbulence based on a nonlinear wave–particle interaction process, called plasma maser effect [2,9–11]. In the present paper also, we intend to attain the same objective based on plasma maser effect, but with a modified formulation considering the nonlinear force involved in it.

Plasma maser effect is one of the lowest order mode–mode coupling processes in plasmas through which transfer of wave energy from resonant wave (low frequency) to nonresonant wave (high frequency) takes place. Plasma maser effect occurs when both resonant and nonresonant modes are present in the system. The resonant modes are those which are in phase relation with thermal particles; for these modes the Cherenkov resonance

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condition $\omega - \mathbf{k}\cdot\mathbf{v} = 0$ is satisfied. On the other hand, the nonresonant modes are those modes for which both the linear and nonlinear conditions are not satisfied, i.e. $\Omega - \mathbf{K}\cdot\mathbf{v} \neq 0$ and $\Omega - \omega - (\mathbf{K}-\mathbf{k})\cdot\mathbf{v} \neq 0$. Here, ω and Ω are the frequencies of the resonant and the nonresonant waves respectively, and $\mathbf{k}$ and $\mathbf{K}$ are the corresponding wave numbers. Plasma maser effect is especially important in a strongly magnetized plasma where $\Omega_e > \omega_{pe}$ [12]. This effect is quite effective as it does not require any frequency matching conditions for the participating waves [10].

Recently, investigation has been carried out to explain physical phenomena related to enhanced electromagnetic fluctuations in space plasmas based on plasma maser effect [13]. In such investigation for inhomogeneous plasma, it is necessary to consider plasma inhomogeneity and the associated density gradient driven low frequency drift wave turbulence.

The physical mechanism of this effect is best explained in terms of high frequency nonlinear force [14,15]. This nonlinear force arises as a result of the resonant interaction between electrons and the modulated field; this modulated field is the result of coupling between the low-frequency wave field with the test high-frequency wave field present in the system. Electrons suffer acceleration (or deceleration) due to this nonlinear force and the accelerated electrons radiate a participating wave. In contrast to the parametric interaction process, the nonlinear force in this mechanism is a high-frequency one and makes the high-frequency wave unstable in the presence of a low-frequency wave.

Earlier the role of nonlinear force in generation of instability in homogeneous plasma has been cleared [14,15]. We, for the first time in this paper, are trying to address this issue for magnetically confined inhomogeneous plasma. Here we have derived the nonlinear force which is responsible for the amplification of ion acoustic wave in magnetically confined inhomogeneous plasmas due to plasma maser effect, and also we are calculating the growth rate of ion acoustic wave using the fluid equations. We have analyzed the role of nonlinear force and elucidated its importance in amplification processes in magnetically confined inhomogeneous plasma. With the derived growth rate, it is also tried to explain for application of plasma maser effect for describing the process of transport of energies within tokamak system.

2. Formulation

We consider a magnetically confined inhomogeneous plasma in presence of drift wave turbulence. The ion-acoustic mode, present in the system, is considered as super imposing perturbation to the system. The confining magnetic field, with negligibly small gradient, is taken along the z-direction $\mathbf{B}_0 = \mathbf{B}_0(y)$ and the system has spatial gradient in y-direction. The geometry of the model is shown in Fig. 1. For such an inhomogeneous plasma [16], the particle distribution function [17] is considered as

$$f_{0j}\left(v_\perp, v_\parallel, \frac{v_x}{\Omega_j} - y\right) \simeq f_{0j}\left(v_\perp^2, v_\parallel\right) + (v_x - \Omega_j y)\frac{1}{\Omega_j}\frac{\partial f_{0j}}{\partial y}, \quad (1)$$

where $f_{0j}(v_\perp^2, v_\parallel)$ is the distribution function for the guiding center; j refers to the electron (e) and ion (i), $v_x = v_\perp \cos\theta$, $v_\parallel$ and $v_\perp$ are the components of velocity along and perpendicular to the direction of the external magnetic field respectively, θ is the phase angle of the particle in the orbit, and $\Omega_j = e_j B_0/mc$ is the electron/ion cyclotron frequency.

We introduce the density gradient parameter as

$$\epsilon' = \left[-\frac{1}{f_{0j}}\frac{\partial f_{0j}}{\partial y}\right]_{y=y_0=0}. \quad (2)$$

Thus, the particle distribution function reduces to the form

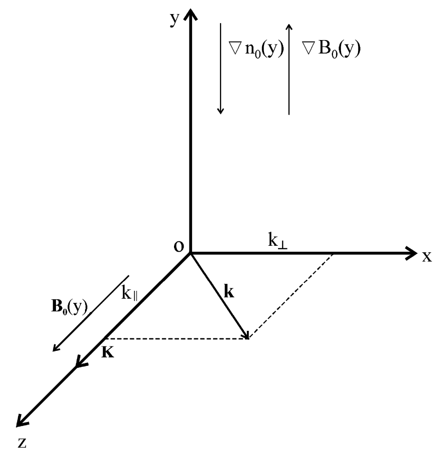

Fig. 1. Geometry of the model; $\mathbf{k} = (k_\perp, 0, k_\parallel)$ is the propagating vector of drift wave, $\mathbf{K} = (0, 0, K_\parallel)$ is the propagating vector of ion acoustic wave, $\nabla n_0(y)$ is the density gradient along the negative y-direction, $\nabla B_0(y)$ is the magnetic gradient along the positive y-direction.

$$f_{0j}\left(v_\perp, v_\parallel, \frac{v_x}{\Omega_j} - y\right) \simeq f_{0j}\left(v_\perp^2, v_\parallel\right)\left\{1 + \epsilon'\left(y - \frac{v_x}{\Omega_j}\right)\right\}. \quad (3)$$

The dynamical state of plasma, due to interaction of high frequency ion acoustic wave with low-frequency drift wave turbulence, is described by the Vlasov equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{e_j}{m_j}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right]F_{0j}(\mathbf{r}, \mathbf{v}, t) = 0. \quad (4)$$

According to the linear response theory [18] of a turbulent plasma, the unperturbed particle distribution function and field are

$$F_{0j} = f_{0j} + \epsilon f_{1j} + \epsilon^2 f_{2j}, \quad (5)$$

and

$$\mathbf{E}_{0e} = \epsilon \mathbf{E}_l + \epsilon^2 \mathbf{E}_2, \quad (6)$$

where ϵ is a small parameter associated with the drift wave turbulent field $\mathbf{E}_l = (E_{l\perp}, 0, E_{l\parallel})$ with propagation vector $\mathbf{k} = (k_\perp, 0, k_\parallel)$, f_{0j} is space and time average part, f_{1j}, f_{2j} are fluctuating parts of the distribution function and E_2 is the second order electric field.

To the order of ϵ, we have from Eq. (4)

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{e_j}{m_j}\left(\frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right]f_{1j} = \frac{e_j}{m_j}\left(\mathbf{E}_l\cdot\frac{\partial}{\partial \mathbf{v}}f_{0j}\right). \quad (7)$$

To find f_{1j}, we use the Fourier transform according to

$$A(\mathbf{r}, \mathbf{v}, t) = \sum_{\mathbf{k},\omega} A(\mathbf{k}, \omega, \mathbf{v})\exp\left[i(\mathbf{k}\cdot\mathbf{r} - \omega t)\right], \quad (8)$$

and following the method of characteristics [17], we have from Eq. (7)

$$f_{1j}(\mathbf{k}, \omega) = i\frac{e_j}{m_j}\left[\frac{m_j}{T_j k_\perp}E_{l\perp}\left\{1 - \left(k_\parallel v_\parallel - \omega - \frac{\epsilon' T_j k_\perp}{m_j \Omega_j}\right)S_{a,b}\right\}f_{0j} - E_{l\parallel}\frac{\partial f_{0j}}{\partial v_\parallel}S_{a,b}\right], \quad (9)$$

where

$$S_{a,b} = \sum_{a,b}\frac{J_a(\alpha_j)J_b(\alpha_j)\exp[i\{b-a\}\theta]}{a\Omega_j + k_\parallel v_\parallel - \omega + i0^+}, \quad \alpha_j = (k_\perp v_\perp)/\Omega_j,$$

and $i0^+$ is a small imaginary part of ω.

Now we perturb the quasisteady state by a high-frequency test ion acoustic wave field $\mu\delta\mathbf{E}_h$ with a propagating vector