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Geodesic mode instability driven by the electron current in tokamak plasmas

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ABSTRACT

Effect of the parallel electron current on Geodesic Acoustic Modes (GAM) in a tokamak is analyzed by kinetic theory taking into the account the ion Landau damping and diamagnetic drifts. It is shown that the electron current modeled by shifted Maxwell distribution may overcome the phase velocity threshold and ion Landau damping thus resulting in the GAM instability when the parallel electron current velocity is larger than the effective parallel GAM phase velocity $Rq\omega$. The instability occurs due to its cross term of the current with the ion diamagnetic drift. Possible applications to tokamak experiments are discussed.

1. Introduction

Geodesic Acoustic Modes (GAM) driven by geodesic plasma compressibility in toroidal geometry [1] have actively been studied theoretically in recent years [1–8]. Oscillations in the geodesic frequency range have been experimentally detected under wide range of condition in various tokamaks, during ohmic, neutral beam (NB), or ion cyclotron resonance heating (ICR) [7–16]. It is expected that these modes will interact and affect stationary plasma rotation [18,19] as well as drift-wave turbulence and plasma transport as has been observed in experiments [16] and simulations [19]. In general, GAM eigen-modes are subdivided into relatively high frequency geodesic mode (GAM) with frequency $\omega_{\mathrm{GAM}}^2 \approx (7T_i/2 + 2T_e)/R_0^2 m_i$ and ion-sound mode $\omega_s^2 \approx T_e/q^2 R_0^2 m_i$ [3–8,20] where R_0 is major radius, q is safety parameter, $T_{e,i}$ electron and ion temperatures. The high frequency modes are often observed as mixed with Alfvén eigenmodes (AE) (known as Alfvén cascades or reversed shear modes, chirping AE, and beta induced modes [7–11]). In T-10 tokamak experiments [14], the oscillations exhibiting GAM features are observed across a substantial part of the minor plasma radius, thus the local frequency of the GAM ω_{GAM} and the ion sound mode ω_s may be close to each other at some radial locations.

It is well known that ion sound instability [21] may be driven by an electric current along magnetic field. Typically, the instability occurs only when the electron beam velocity is higher than the ion sound velocity ($v_0 > c_s$) and the electron temperature is much larger than the ion temperature ($T_e \geqslant 6T_i$). Typically, a high electron current velocity may appear during current rump-up state or/and during a counter NB injection in tokamaks [10,15,22].

Here, we investigate whether the parallel electron current can drive the GAM modes via coupling with the drift dynamics. We present the kinetic treatment of the GAM type modes by fully taking into account parallel electron and ion dynamics and the ion diamagnetic drifts. Drift effect on GAM type modes was studied before [2,3,7,8], but the electron current was not included. A general kinetic treatment of geodesic eigenmodes is rather difficult, but the procedure is simplified for the geodesic continuum (GC). We do not consider here the eigen-mode structure, we consider only the continuum modes, similarly to other papers [3,4,6,7,17].

2. Dispersion equation

We employ the quasi-toroidal set of coordinates (r, ϑ, ζ) in the large aspect ratio tokamak approximation [9] $R_0 \gg r$, where the circular surfaces ($R = R_0 + r\cos\vartheta$, $z = r\sin\vartheta$) are formed by the magnetic field with toroidal and poloidal components, $B_\zeta = B_0 R_0/R$, $B_\vartheta = rB_\zeta/qR_0$, and $B_\vartheta \ll B_\zeta$. The cylindrical coordinate system is used in the velocity space, where $\{v_\perp, \sigma, v_\parallel\}$ are, respectively, the perpendicular, angular, and parallel components with respect of the magnetic field. The standard approach of small Larmor

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radius is assumed to obtain a drift kinetic equation [21,23] in the form

$$\frac{\partial f}{\partial \vartheta}-\frac{\mathrm{i}\Omega f}{w}=\frac{eF}{mw}\left[\frac{(w-v_0)E_3}{k_0 v_T^2}+\frac{2+\eta(w^2+u^2-3)}{2k_0 v_T\omega_c d_r}E_2 - \frac{(2w^2+u^2)}{2v_T\omega_c k_0 R}E_1\sin\vartheta\right]. \quad (1)$$

Here $\Omega=\omega/k_0 v_{Ti}$ and $\omega_c=eB/mc$ are the normalized wave and cyclotron frequencies, E_i are components of wave field where the parallel field E_3 is potential part of electromagnetic field, $E_3=h_\theta E_2$, $h_\vartheta=B_\vartheta/B_0$ is magnetic field inclination, $k_0=h_\vartheta/r$ is the parallel wave vector, $v_{Te,i}=\sqrt{T_{e,i}/m_{e,i}}$ are thermal velocities, $\partial n_0/\partial r=-n_0/d_r$ is density gradient, $\eta_{e,i}=\partial\ln T_{e,i}/\partial\ln n_0$, $w=v_\parallel/v_T$ and $u=v_\perp/v_T$ are normalized space velocities. For the $N=0$ toroidal mode number $k_0=1/qR$. Maxwell distribution function F is assumed for ions and the electron distribution is Maxwell distribution shifted by the parallel current velocity V_0, $F_e=F(1+v_\parallel V_0/v_{Te}^2)$. We use the potential approximation for the electric fields neglecting the magnetic perturbation. Integrating Eq. (1) for electrons in the limit $\omega\ll v_{Te}/qR$ ($\Omega\ll v_{Te}/v_{Ti}$ in our notation) and $V_0/v_{Te}=\mu v_0\ll 1$, we obtain the equations for the electron density perturbations $\tilde{n}_e$

$$n_s=\frac{e_i n_0 R_0 q}{T_e}\times\left[\sqrt{\frac{\pi}{2}}\mu\big(v_0+t_e\rho(1-\eta_e/2)\big)E_s-\left(1+\mathrm{i}\sqrt{\frac{\pi}{2}}\mu\Omega\right)E_c\right],$$
$$n_c=\frac{e_i n_0 R_0 q}{T_e}\times\left[\sqrt{\frac{\pi}{2}}\mu\big(v_0+t_e\rho(1-\eta_e/2)\big)E_c+\left(1+\mathrm{i}\sqrt{\frac{\pi}{2}}\mu\Omega\right)E_s\right] \quad (2)$$

where $\mu=v_{Ti}/v_{Te}$, $E_3=E_s\sin\vartheta+E_c\cos\vartheta$, $\tilde{n}_e=n_s\sin\vartheta+n_c\cos\vartheta$ and the contribution of small electron diamagnetic drift velocity was combined with the contribution from the parallel current: $w_0=v_0+t_e\rho(1-\eta_e/2)$, where $\rho=v_{Ti}/d_r\omega_{ci}h_p$ is the normalized drift parameter. Then, using the radial magnetic drift velocity, we get the radial component of the electron current

$$\tilde{j}_r^e=e\frac{v_{Te}^3}{2}\int_0^\infty u\,du\oint d\vartheta\int_{-\infty}^{\infty}V_{re}f_e\,dw = \frac{e_i^2 q n_0}{m\omega_{ci}}\times\left[\mathrm{i}\sqrt{\frac{\pi}{2}}\frac{\mu}{2}\left(w_0+t_e\frac{\rho}{2}(2+\eta_e)\right)E_s-\left(1+\mathrm{i}\mu\sqrt{\frac{\pi}{2}}\frac{\Omega}{2}\right)E_c\right] \quad (3)$$

where $V_{re,i}=-(2w^2+u^2)v_{Te,i}^2\sin\vartheta/2R\omega_{ce,i}$, and $t_e=T_e/T_i$. Using $v_0=0$ in Eq. (1) for ions, we get the equations for the $\sin\theta$- and $\cos\theta$-components of the perturbed distribution function

$$f_s=\frac{\mathrm{i}e_i\Omega qRFE_s}{T_i(\Omega^2-w^2)}\left(w+\rho+\eta\rho\frac{(w^2+u^2-3)}{2}\right)-\frac{e_iqRFE_c}{T_i(\Omega^2-w^2)}\left(w^2+w\rho+w\rho\eta\frac{(w^2+u^2-3)}{2}\right)-\frac{\mathrm{i}e_i\Omega q(2w^2+u^2)FE_1}{2\omega_{ci}m_iv_{Ti}(\Omega^2-w^2)}, \quad (4a)$$

$$f_c=\frac{e_iqRwFE_s}{T_i(\Omega^2-w^2)}\left(w+\rho+\rho\eta\frac{(w^2+u^2-3)}{2}\right)+\frac{\mathrm{i}e_i\Omega qRFE_c}{T_i(\Omega^2-w^2)}\left(w+\rho+\eta\rho\frac{(w^2+u^2-3)}{2}\right)-\frac{e_iqw(2w^2+u^2)FE_1}{2\omega_{ci}m_iv_{Ti}(\Omega^2-w^2)}. \quad (4b)$$

These expressions define the ion density perturbations as follows:

$$n_{ic}=-\frac{e_in_0R_0q}{2m_iv_{Ti}^2}\times\left[(2+\sqrt{2}\Omega Z)E_s+\mathrm{i}\left(\frac{\sqrt{2}}{2}(2+\Omega^2\eta-\eta)Z+\Omega\eta\right)\rho E_c\right], \quad (5a)$$

$$n_{is}=\frac{e_in_0R_0q}{2m_{i0}v_{Ti}^2}\left\{\frac{\mathrm{i}v_{Ti}}{R_0\omega_{ci}}\big(\sqrt{2}(\Omega^2+1)Z/2+\Omega\big)E_1+\big[(2+\sqrt{2}\Omega Z)E_c-\mathrm{i}\big(\sqrt{2}(2+\Omega^2\eta-\eta)Z/2+\Omega\eta\big)\rho E_s\big]\right\}. \quad (5b)$$

Using the quasi-neutrality condition with the electron density from Eq. (2), one finds the electric field components in the form

$$E_s=\mathrm{i}\frac{\{\sqrt{2\pi}\mathrm{i}\mu w_0-\lfloor\sqrt{2}(\Omega^2\eta+2-\eta)Z/2+\eta\Omega\rfloor t_e\rho\}}{2[1+t_e(1+\sqrt{2}\Omega Z/2)+\sqrt{2\pi}\mathrm{i}\mu\Omega/2]}E_c, \quad (6a)$$

$$E_c=-\mathrm{i}\frac{t_ev_{Ti}[\Omega+\sqrt{2}(1+\Omega^2)Z/2]}{R_0\omega_{ci}D}\times\left[1+t_e\left(1+\sqrt{2}\Omega\frac{Z}{2}\right)+\frac{\sqrt{2\pi}}{2}\mathrm{i}\mu\Omega\right]E_1 \quad (6b)$$

where

$$D=\big[(\sqrt{2}Z/2+1)t_e+\sqrt{2\pi}\mathrm{i}\mu\Omega/2+1\big]^2-\frac{1}{4}\{[\sqrt{2}((\Omega^2-1)\eta/2+1)Z+\Omega\eta]t_e\rho-\mathrm{i}\sqrt{2\pi}\mu w_0\}^2. \quad (7)$$

The ion radial geodesic current is found from the ion distribution function in Eqs. (4a), (4b)

$$j_{ri}=\frac{e_i^2n_0q}{4m_i\omega_c}\left\{\mathrm{i}\rho\big[\sqrt{2}(\eta\Omega^4+2\Omega^2+2+\eta)Z/2+\eta\Omega^3+\Omega(2+\eta)\big]E_s-\big[\sqrt{2}\Omega(\Omega^2+1)Z+2\Omega^2+4\big]E_c-\frac{\mathrm{i}v_{Ti}}{R_0\omega_c}\big[(\Omega^4+2\Omega^2+2)Z+2\Omega(\Omega^2+3)\big]E_1\right\}. \quad (8)$$

The final equation for the geodesic continuum, which is obtained from the quasi-neutrality condition $\tilde{j}_r^e+\tilde{j}_r^i+j_p=0$, where j_p is the ion radial polarization current $j_p=-\mathrm{i}\omega c^2E_1/4\pi c_A^2$, $c_A=B/\sqrt{4\pi n_im_i}$ has the form

$$\Omega^2/q^2=\Psi/D \quad (9)$$

where Ψ is given by the expression

$$\Psi=-D\big[\sqrt{2}(\Omega^4/2+\Omega^2+1)Z+\Omega(\Omega^2+3)\big]\Omega/2+t_e\frac{\Omega}{2}\big[\sqrt{2}(\Omega^2+1)Z/2+\Omega\big]\times\Big\langle\Omega\big[1+(\sqrt{2}\Omega Z/2+1)t_e+\sqrt{2\pi}\mathrm{i}\mu\Omega/2\big]\times\big[\sqrt{2}(\Omega^2+1)Z/2+\Omega-\sqrt{2\pi}\mathrm{i}\mu/2\big]$$