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Decagonal quasicrystal plate with elliptic holes subjected to out-of-plane bending moments

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ABSTRACT

In the present paper, we consider only the ideal elastic behavior, neglecting the dissipation associated with the atomic rearrangements. Under these conditions, the decagonal quasicrystal plate bending problems have been discussed. The Stroh-like formalism for the bending theory of decagonal quasicrystal plate is developed. The analytical solutions for problems of decagonal quasicrystal plate with elliptic hole subjected to out-of-plane bending moments are obtained directly by using the forms. The resultant bending moments around the hole boundaries are also given explicitly. When the phonon–phason coupling is absent, the results reduce to the corresponding solutions for the isotropic elastic plates.

1. Introduction

After the first discovery of a quasicrystalline phase (icosahedral structure) by Shechtman in 1984 [1], much research was performed on the electronic structure and the optic, magnetic, thermal and mechanical properties of quasicrystals [2]. Elasticity is one of the important properties of quasicrystals. The elastic behavior of quasicrystals is different from that of usual crystals. Based on Landau–Anderson symmetry-breaking, the phason as a new elementary excitation was introduced in addition to the well known phonon. Phonons are responsible for translations of particles while phasons are responsible for rearrangements of local atomic configurations [3–5].

The problems of quasicrystals containing holes and cracks have been studied extensively for two-dimensional deformations [6,7]. Many methods and techniques have been developed to solve problems of elasticity and defects in quasicrystals. Among them, the decomposition procedure, the Green function method and integral transformations have been particularly successful [8–11]. However, relatively little work involving the bending problems of quasicrystals has been done due to the mathematical complexity. Boundary conditions for plate bending in one-dimensional hexagonal quasicrystals and two-dimensional dodecagonal quasi-crystal have been considered by Gao et al. [12,13].

Recently, the complex variable methods in quasicrystal elasticity have reached a big step by connecting Muskhelishvili method, Lekhnitskii formulation and Stroh formalism [14–16]. However, still very few contributions have been made to the plate bending problems. The Stroh formalism is an elegant and powerful tool for the study of two-dimensional deformation of quasicrystal materials, which has been applied successfully to solve the elliptical hole, the rigid-line inclusion problems and the interaction between defects [17]. Under the conditions (as already mentioned), Stroh-like formalism for the bending theory of quasicrystal plates is developed in this paper. In our formalism, the deflections, the moments and the transverse shear forces can all be expressed in complex matrix form. Based on the developed formalism, the analytical solutions for decagonal plates with elliptic hole subjected to out-of-plane bending moments are now obtained explicitly. The solutions for the crack problems are obtained by letting the minor axis of the ellipse approach to zero and the moment intensity factors of the cracks are also given. Furthermore, a numerical example is given for the reader to make a quantitative assessment.

2. Stroh-like formalism for the bending theory of decagonal quasicrystal plate

The drawbacks in standard format of quasicrystal linear elasticity have been discussed in Refs. [18–20]. In principle, a conservative

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component of the inner self-action may exist, which make analysis more difficult. Following the point of view adopted in [21], here we adopt generalized elastic constitutive prescriptions [5] and the common assumption is accepted. The restriction of the analysis to the linear elastic case suggests to attribute an ideal limit character to the present results [21].

Assume that decagonal quasicrystal is periodic in z direction, and quasi-periodic in the x–y plane. According to the elastic theory of quasicrystals [5,6], we have the strain-displacement relations

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad w_{ij} = \frac{\partial w_i}{\partial x_j} \tag{1}$$

the equilibrium equations

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0, \qquad \frac{\partial H_{ij}}{\partial x_j} = 0 \tag{2}$$

and the constitutive equations

$$\begin{aligned}
&\sigma_{xx} = C_{11}\varepsilon_{xx} + C_{12}\varepsilon_{yy} + R(w_{xx} + w_{yy})\\
&\sigma_{yy} = C_{12}\varepsilon_{xx} + C_{11}\varepsilon_{yy} - R(w_{xx} + w_{yy})\\
&\sigma_{zz} = C_{13}\varepsilon_{xx} + C_{13}\varepsilon_{yy} + C_{33}\varepsilon_{zz}\\
&\sigma_{xy} = \sigma_{yx} = 2C_{66}\varepsilon_{xy} + R(w_{yx} - w_{xy})\\
&\sigma_{yz} = \sigma_{zy} = 2C_{44}\varepsilon_{yz}\\
&\sigma_{xz} = \sigma_{zx} = 2C_{44}\varepsilon_{xz}\\
&H_{xx} = K_1 w_{xx} + K_2 w_{yy} + R(\varepsilon_{xx} - \varepsilon_{yy})\\
&H_{yy} = K_1 w_{yy} + K_2 w_{xx} + R(\varepsilon_{xx} - \varepsilon_{yy})\\
&H_{xy} = K_1 w_{xy} - K_2 w_{yx} - 2R\varepsilon_{xy}\\
&H_{yx} = K_1 w_{yx} - K_2 w_{xy} + 2R\varepsilon_{xy}\\
&H_{xz} = K_3 w_{xz}\\
&H_{yz} = K_3 w_{yz}
\end{aligned} \tag{3}$$

in which $C_{66} = (C_{11} - C_{12})/2$. σ_{ij} ($\sigma_{ij} = \sigma_{ji}$), ε_{ij} ($\varepsilon_{ij} = \varepsilon_{ji}$), u_i and C_{ij} are the stress, strain, displacement, and elastic constants of phonon fields, respectively. H_{ij} ($H_{ij} \neq H_{ji}$), w_{ij} ($w_{ij} \neq w_{ji}$), w_i and K_i are the stress, strain, displacement, and elastic constants of phason fields. R is the phonon–phason coupling elastic constant.

The approximate theory of bending of decagonal quasicrystal plates (thin plates) is based on the Kirchhoff plate assumptions. It follows from the assumption that

$$\begin{aligned}
&u_x = -z\frac{\partial w}{\partial x}, \qquad u_y = -z\frac{\partial w}{\partial y}\\
&w_x = -z\frac{\partial u}{\partial x}, \qquad w_y = -z\frac{\partial v}{\partial y}
\end{aligned} \tag{4}$$

where $w(x, y)$ is the deflection of the middle plane, $u(x, y)$ and $v(x, y)$ are the generalized deflection of the middle plane. By Eqs. (1) and (4), we have

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2}, \qquad \varepsilon_{yy} = -z\frac{\partial^2 w}{\partial y^2}, \qquad \varepsilon_{xy} = -z\frac{\partial^2 w}{\partial x \partial y} \tag{5}$$

$$\begin{aligned}
&w_{xx} = -z\frac{\partial^2 u}{\partial x^2}, \qquad w_{yy} = -z\frac{\partial^2 v}{\partial y^2}\\
&w_{xy} = -z\frac{\partial^2 u}{\partial x \partial y}, \qquad w_{yx} = -z\frac{\partial^2 v}{\partial x \partial y}
\end{aligned} \tag{6}$$

When we cut this plate with certain surfaces parallel to the initial middle surface xy with height equal to the plate thickness h, then the bending moments M_{xx}, M_{yy} and M_{xy}, the generalized bending moments N_{xx}, N_{xy}, N_{xy} and N_{yx} can be expressed as follows

$$\begin{aligned}
&M_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx} z\,dz, \qquad M_{yy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{yy} z\,dz\\
&M_{xy} = M_{yx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} z\,dz
\end{aligned} \tag{7}$$

$$\begin{aligned}
&N_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} H_{xx} z\,dz, \qquad N_{yy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} H_{yy} z\,dz\\
&N_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} H_{xy} z\,dz, \qquad N_{yx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} H_{yx} z\,dz
\end{aligned} \tag{8}$$

Substituting Eqs. (5) and (6) into Eqs. (3) then into Eqs. (7)–(8), the bending moments and the generalized bending moments can be expressed as

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \\ N_{xx} \\ N_{yy} \\ N_{xy} \\ N_{yx} \end{Bmatrix} = -\mathbf{D} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \\ \frac{\partial^2 u}{\partial x^2} \\ \frac{\partial^2 v}{\partial y^2} \\ \frac{\partial^2 u}{\partial x \partial y} \\ \frac{\partial^2 v}{\partial x \partial y} \end{Bmatrix} \tag{9}$$

in which the matrix $\mathbf{D}$ (see Appendix A) is bending stiffness matrix.

We consider now the case when the plate is subjected to bending only by forces and moments distributed along the edge. The force and moment equilibrium equations of the plate can be expressed as

$$\begin{aligned}
&\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = 0, \qquad \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0\\
&\frac{\partial M_{yx}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = 0
\end{aligned} \tag{10}$$

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \qquad \frac{\partial N_{yx}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0 \tag{11}$$

in which Q_x and Q_y are the transverse shear forces defined by

$$Q_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xz}\,dz, \qquad Q_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{yz}\,dz$$

The equilibrium equations (10) and (11) can be satisfied automatically if we introduce stress functions $\psi_1(x, y)$, $\psi_2(x, y)$, $\phi_1(x, y)$ and $\phi_2(x, y)$ such that

$$\begin{aligned}
&M_{xx} = -\frac{\partial \psi_1}{\partial y}, \qquad M_{yy} = \frac{\partial \psi_2}{\partial x}, \qquad M_{xy} = \frac{1}{2}\left(\frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial y}\right)\\
&N_{xx} = -\frac{\partial \phi_1}{\partial y}, \qquad N_{yy} = \frac{\partial \phi_2}{\partial x}\\
&N_{xy} = \frac{\partial \phi_1}{\partial x}, \qquad N_{yx} = -\frac{\partial \phi_2}{\partial y}
\end{aligned} \tag{12}$$