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# Josephson current through a T-shaped double quantum dots: $\pi$-junction transition and interdot antiferromagnetic correlations

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ABSTRACT

By means of the exact diagonalization approach, the Josephson current in a T-shaped double quantum dot structure is theoretically investigated. The ground state is obtained within zero bandwidth approximation in which the superconductors are replaced by effective local pairing potentials. It is found that Josephson current can flow through this structure in the presence of various electron correlations. Furthermore, in the half-filled case, a novel 0–$\pi$ transition behavior is observed, which arises from the interplay of interdot antiferromagnetic coupling and electron correlations.



## 1. Introduction

The continuous progress in nanofabrication during the last decades made it possible to investigate basic physical effects in a very controlled manner. One example of such highly controllable devices is quantum dot (QD), which is used, amongst various applications, for a detailed and very controlled study of the Kondo effect [1–3].

In comparison with a single QD structure, the coupled double-quantum-dot (DQD) systems provide much more Feynman paths [4] for the electron transmission and possess more tunable parameters to manipulate the electronic transport behaviors [5–10]. In these systems, the interdot electron correlation plays an important role in determining the spin configuration of the ground state. For instance, in serial DQD system, the localized moments on the QDs either form the Kondo singlet with the conduction electrons in the leads or form a local spin singlet, dependent on the interdot hopping [9]. In parallel DQD structures, recent experiments demonstrated that an extraordinary control over the physical properties of DQDs can be achieved, which enables the direct experimental investigations of the competition between the Kondo effect and the interdot exchange interaction between localized moments on the dots [10].

When the leads of the single QD systems are superconductors, the electron correlations can also induce various interesting phenomena, due to the interplay between the Josephson effect and electron correlations [11–17]. A paradigmatic example is the prediction about a transition to $\pi$ junction behavior as a function of some relevant system parameters [14]. In these systems, the ratio of the Kondo temperature $T_K$ to the superconducting gap $\Delta$ is a key parameter. In the strong coupling limit $T_K \gg \Delta$, the Kondo effect survives even in the presence of the superconductivity; a Cooper pair is broken in order to screen the localized spin in the QD. On the other hand, in the weak coupling limit $T_K \ll \Delta$, the Kondo effect is negligible because a strongly bound Cooper pair cannot be broken. Then, the Cooper pair feels the localized magnetic moment in the QD. Under this situation, when Coulomb interaction is strong inside the QD, the so-called 0–$\pi$ transition occurs. To be concrete, the dependence of Josephson current $I_J$ on the macroscopic phase difference $\varphi$ between the superconductors changes from $I_J = I_c \sin(\varphi)$ to $I_J = I_c \sin(\pi + \varphi) = -I_c \sin(\varphi)$, and then the critical current $I_c$ becomes negative. But, in the case of coupled-DQD geometries, the situation is more complicated due to the interplay among the Kondo effect, the interdot exchange interaction, and pairing correlations [18–20]. In this case, there are three different energy scales, namely, Kondo temperature $T_K$, interdot antiferromagnetic exchange interaction $J$, and superconducting gap $\Delta$. Accordingly, this system exhibits a richer magnetic behavior than the single-QD case, due to the competition between Kondo effect and the other two kinds of electron correlations. Recently, R. Žitko et al. investigated the Josephson current through a serial DQDs by use of the numerical renormalization group (NRG) approach [20]. They found that there exists a rich phase diagram of the 0 and $\pi$-junction regimes by adjusting the interdot exchange interaction $J$. In particular, when both the superconductivity and the exchange interaction compete with the Kondo physics

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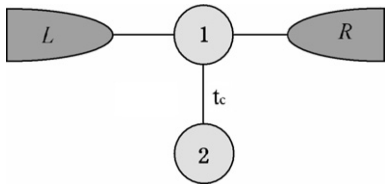


**Fig. 1.** Schematic of a S/TDQD/S structure. QD-1 is connected to two s-wave BCS superconductor leads and QD-2 is side coupled to QD-1 with interdot coupling parameter $t_c$.

($\Delta \sim J \sim T_K$), there appears an island of $\pi'$ phase at large values of the superconducting phase difference.

In this work, we investigate the Josephson current in a T-shaped DQDs structure by means of exact diagonalization techniques. The structure that we consider is illustrated in Fig. 1, and hereafter the system is referred to as S/TDQD/S. QD-1 is connected to two s-wave Bardeen–Cooper–Schrieffer (BCS) superconductor leads and QD-2 is side coupled to QD-1 with interdot coupling parameter $t_c$. By adjusting the relevant parameters of such a structure, we observed a rich phase diagram of the 0–$\pi$ transition caused by the competition among the Kondo effect, the interdot exchange interaction, and pairing correlations. Moreover, we find that there is a novel 0–$\pi$ transition behavior originating from the interplay between the interdot antiferromagnetic coupling and electron correlations.

The rest of this Letter is organized as follows. In Section 2, we introduce the model Hamiltonian of the system and the method of calculation. The numerical results are presented and discussed in Section 3. In Section 4, we give the summary.

## 2. Model and method

In order to describe the system shown in Fig. 1, we use the following Hamiltonian, which can be written as:

$$H = \sum_{\alpha=L,R} H_\alpha + H_D + H_T, \tag{1}$$

with

$$\begin{aligned} H_\alpha &= \sum_{k,\sigma} \varepsilon_{\alpha,k} a^\dagger_{\alpha,k\sigma} a_{\alpha,k\sigma} \\ &\quad + \sum_k \left(\Delta e^{i\varphi_\alpha} a_{\alpha,k\downarrow} a_{\alpha,-k\uparrow} + \Delta e^{-i\varphi_\alpha} a^\dagger_{\alpha,-k\uparrow} a^\dagger_{\alpha,k\downarrow}\right), \\ H_D &= \sum_{\sigma,j=1}^{2} \varepsilon_j d^\dagger_{j\sigma} d_{j\sigma} + \sum_\sigma \left(t_c d^\dagger_{2\sigma} d_{1\sigma} + \text{H.c.}\right) + \sum_{j=1}^{2} U_j n_{j\uparrow} n_{j\downarrow}, \\ H_T &= \sum_{k,\sigma} \left(V_{L,k} a^\dagger_{L,k\sigma} d_{1\sigma} + V_{R,k} a^\dagger_{R,k\sigma} d_{1\sigma} + \text{H.c.}\right). \end{aligned} \tag{2}$$

In the above equations, $H_\alpha$ ($\alpha = L, R$) is the standard BCS mean-field Hamiltonian for the superconducting leads with phase $\varphi_\alpha$ and energy gap $\Delta$. The chemical potentials of both leads are set as zero. $H_D$ models the T-shaped DQDs. And, $H_T$ denotes the tunneling part of the Hamiltonian between lead-$L(R)$ and QD-1. $t_c$ is the interdot coupling coefficient. On the other hand, $a^\dagger_{\alpha,k\sigma}$ and $d^\dagger_{j\sigma}$ ($a_{\alpha,k\sigma}$ and $d_{j\sigma}$) are operators to create (annihilate) an electron with momentum $k$ and spin orientation $\sigma = \uparrow (\downarrow)$ in lead-$\alpha$ and in the $j$th QD, respectively. $\varepsilon_{\alpha,k}$ and $\varepsilon_j$ denote the corresponding energy levels. $U_j$ indicates the strength of intradot Coulomb repulsion in the corresponding QD. $V_{\alpha,k}$ denotes the coupling between lead-$\alpha$ and QD-1. Note that the determination of the ground state properties of this model is a formidable task, which requires some approximation scheme. A great simplification can be made by integrating out the electronic degrees of freedom of the superconducting leads [19]. This procedure leads to an effective low energy theory in which each superconductor is replaced by a single site with an effective pairing potential $\tilde{\Delta}$. Also, the hopping terms between the leads and the QD are replaced by an effective parameter $\tilde{V}_\alpha$. Then, we can write the new expressions of $H_\alpha$ and $H_T$, i.e.,

$$\begin{aligned} H_\alpha &= \sum_\sigma \varepsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} + \tilde{\Delta} e^{i\varphi_\alpha} a_{\alpha,\downarrow} a_{\alpha,\uparrow} + \tilde{\Delta} e^{-i\varphi_\alpha} a^\dagger_{\alpha,\uparrow} a^\dagger_{\alpha,\downarrow}, \\ H_T &= \sum_\sigma \left(\tilde{V}_L a^\dagger_{L,\sigma} d_{1\sigma} + \tilde{V}_R a^\dagger_{R,\sigma} d_{1\sigma} + \text{H.c.}\right). \end{aligned} \tag{3}$$

This approach, usually referred to as the zero bandwidth model (ZBWM), has been discussed for some previous studies [18,19]. One can see that the ZBWM can give qualitatively correct results and can grasp the ground state properties in this kind of systems in the approximate range of $\Gamma \leqslant \Delta$, where $\Gamma$ is the standard tunneling rate to the leads.

We should mention that the Hilbert space of this S/TDQD/S system within the ZBWM is restricted to $4^4$ states and the $z$ component of the total spin $S$ is a good quantum number. Thus, the eigenstates can be characterized in terms of $S_z$ and the eigenenergies can be obtained by block diagonalization of the Hamiltonian matrix. In the superconducting case we distinguish four different ground states, i.e., the pure 0 and $\pi$ states (for these states, the ground state energy as a function of the superconducting phase difference $\varphi = \varphi_L - \varphi_R$ has a global minimum at the points of $\varphi = 0$ and $\pi$), and two intermediate $0'$ and $\pi'$ phases (they are both local minima and dependent on whether $\varphi = 0$ or $\varphi = \pi$ is global minimum [11]).

The Josephson current flowing through the S/TDQD/S system at zero temperature can be obtained by deriving the ground state energy with respect to the phase difference, i.e.,

$$I_J = \frac{2e}{\hbar} \frac{\partial E(\varphi)}{\partial \varphi}. \tag{4}$$

## 3. Results and discussion

By using the formulas developed in Section 2, we perform a numerical calculation to investigate the characteristics of the Josephson currents in the S/TDQD/S structure. In this Letter, we consider that temperature is zero and $\tilde{\Delta} = 1$, i.e., all the energy quantities are scaled by $\tilde{\Delta}$. In principle, the effective parameters $\tilde{\Delta}$ and $\tilde{V}_\alpha$ in this approach have to be determined from the bare parameters $\Delta$ and $V_\alpha$ by means of a self-consistency condition and using a renormalization group analysis, as discussed by Affleck et al. in a previous paper [21]. However, we shall adopt here the simplified assumption that $\tilde{\Delta} = \Delta = 1$ and $\tilde{V}_L = \tilde{V}_R = 1$ without an attempt to fine tune them within the range of parameters considered. This is a reasonable choice as far as we are interested in the qualitative trends rather than in the detailed quantitative results. To get the main physical insight, we set the Fermi energy of leads to be zero. And only the case of symmetric QD-lead coupling is considered with $\tilde{V}_L = \tilde{V}_R = 1$. Furthermore, we set $\varphi_L = -\varphi_R = \varphi/2$, $\varepsilon_j = \varepsilon_0$ and $t_c = 6$ unless otherwise specified.

Fig. 2(a) shows a $(\varepsilon_0, U)$ phase diagram for the 0–$\pi$ transition of the Josephson current in this S/TDQD/S structure. For comparison, the 0–$\pi$ transition phase diagram of the serial DQDs system with the same parameters is also shown in the inset. In this figure, one can clearly see that there are three 0–$\pi$ transition regions for the T-shaped structure but only two transition regions for the serial one. In order to further investigate the mechanism of phase transition, we consider the case of $U = 15$ and plot the total mean charge per spin, namely $N_{e\uparrow}$ and $N_{e\downarrow}$ as functions of QD level $\varepsilon_0$. The numerical results are shown in Fig. 2(b). The total