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Parameter estimation in dynamic Casimir force measurements with known periodicity

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ABSTRACT

It is important to have an accurate estimate of the unknown parameters such as the separation distance between interacting materials in Casimir force measurements. Current methods tend to produce large estimation errors. In this Letter, we present a novel method based on an adaptive control approach to estimate the unknown parameters using large amplitude dynamic Casimir measurements at separation distances of below 1 μm where both electrostatic force and Casimir force are significant. The estimate is proved to be accurate and the effectiveness of our method is demonstrated via a numerical example.

1. Introduction

It was predicted that an attractive force exists between two uncharged parallel metallic plates placed in vacuum without external electromagnetic fields [1]. This effect is called Casimir effect which can be explained by quantum mechanics that the imposed metallic boundaries change the zero point electromagnetic energy between the two plates [2]. The concept of Casimir effect is fundamental in modern physics and it is related to elementary particle physics, gravitation and cosmology, atomic and condensed matter physics [3]. The Casimir effect is a macroscopic quantum effect and the Casimir force becomes strong when the separations between interacting materials are in the nanometer scale [4]. This makes the measurement of the Casimir force possible by using some microelectromechanical systems (MEMS) [5] and atomic force microscopy (AFM) [6]. Besides its impact on fundamental physics, the Casimir force at the nanometer scale is playing a big role in MEMS technology and it will cause stiction and device failure if the separation gaps among components are less than certain values [7]. Over the last decade, extensive experiments have been performed to study the effect of surface roughness, dielectric permittivity of materials and temperature on Casimir force [8]. Most Casimir force measurements are conducted between a sphere and a plate to avoid the difficulty in maintaining a high degree of parallelism between plates [9]. The Casimir force is nonlinearly dependent on the separation between interacting materials. It is easy to compare theories and experimental results by measuring the strength of the Casimir force at various separations. However, these static measurements are vulnerable to noise and drift [10]. It has been proven that dynamic measurement is a better choice for weak force measurements [11]. It has been applied to Casimir force measurements [6] and the measurement sensitivity will be greatly improved if the vibration amplitude is enlarged [12].

Currently, a crucial challenge in Casimir force measurements is the ability to obtain an accurate estimate of the separation gap between two interacting materials, as have been pointed out by several research groups [13,14]. The reason is that the Casimir force is detectable only when the separation gap is less than several micrometers which is much smaller than the size of the interacting materials. This creates difficulty if external devices are employed to measure this quantity. Currently, the separation gap is estimated during the electrostatic calibration in most Casimir force measurements [14,15]. Even if the interacting materials are clean and the measurement is conducted in vacuum environment, electrostatic force exists besides the Casimir force. The strength of the electrostatic force is dominating when the separation gap is above 1–2 μm and the electrostatic calibration is conducted in this range before the Casimir force measurements at smaller separations. The electrostatic force has the expression of $F_e = -\pi\epsilon_0 R[V_m^2 + V_1^2]/d$ where ϵ_0 is the permittivity of free space, V_m is the contact potential difference between two interacting materials, V_1 is due to the electrostatic patch charge and d is the separation gap. In electrostatic calibrations, a bias voltage is tuned between interacting materials to zero V_m at a fixed d by minimizing the electrostatic

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force. Parabolic fitting is used to estimate d [16]. There are several challenges to obtain an accurate estimate of d via this method. First of all, the sphere radius R is determined by Scanning Electron Microscopy (SEM). Even if the measurement error in R is only 1%, the resulting estimation error in d is more than 10 nm. This results in significant measurement errors because some Casimir force measurements are conducted when d is less than 100 nm [6]. Secondly, the estimation accuracy is further degraded by drift and noise since the electrostatic calibration is conducted when the sphere is kept static and it is time consuming for the voltage tuning process.

To improve the accuracy in separation gap estimation, it is desired to conduct the estimation process in large amplitude dynamic force measurements at separations of below 1 μm. But several new challenges are created which make an accurate estimation difficult. Both the Casimir force and the electrostatic force need to be taken into account when d is below 1 μm. In addition, nonlinear dynamics like hysteresis, bistability and harmonics may be observed if the vibration amplitude of the sphere is large because both the electrostatic force and the Casimir force are highly nonlinear when d is small [5,17,18]. From system control point of view, the unknown parameter d is nonlinearly parameterized in the dynamic system and inaccurate estimates will be obtained if conventional parameter estimation techniques like linear filtering [19] are applied. Although several adaptive control approaches have been proposed for the purpose of parameter estimation in dynamic systems with nonlinearly parameterized unknown parameters [20–22], they are not suitable for our problem because the assumptions made in their methods are not valid here. In this Letter, we will propose a novel approach based on periodic adaptation [23] to estimate the interaction forces. Then a new parameter estimation technique is introduced to estimate the unknown separation gap. We prove that the estimate of the separation gap will converge to the true value eventually. To facilitate the real implementation of our method in practice, all the assumptions made in our method are compatible with the real set-up and only the measurable data is processed in the estimation. Last but not least, the effectiveness of our method is shown via a numerical example.

2. Problem formulation

The algorithm proposed is based on the dynamic Casimir force measurements using AFM. The setup consists of a polystyrene sphere attached on a cantilever (Fig. 1). The piezo is used to adjust the separation between the sphere and the sample. A sinusoidal electrical signal is used to excite the cantilever vibration and the sphere dynamics is recorded via an optical detector. The system dynamics of the sphere can be described as [24]:

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -\mu x_2 - kx_1 + u + F(R, x_1 + z) \quad (1)$$

where $x = [x_1, x_2]^T$ are the states of the system and they are measurable. They describe the displacement and velocity of the sphere movement. $\mu > 0$ is the normalized damping of the cantilever and $k > 0$ is the normalized spring constant of the cantilever. μ and k can be measured during the calibration. u is the electrical excitation signal which is continuous and bounded, $F(R, x_1 + z)$ denotes the normalized electrostatic force and the Casimir force between the sphere and the sample and it is formulated as:

$$F(R, x_1 + z) = -\frac{R\pi\epsilon_0 V_1^2}{m(x_1 + z)} - \frac{R\pi^3\hbar c}{360m(x_1 + z)^3} r_c(x_1 + z) \quad (2)$$

where the first term is the residue electrostatic force after the electrostatic calibration and V_1 can be calculated during the process. The second term is the Casimir force where m is the effective

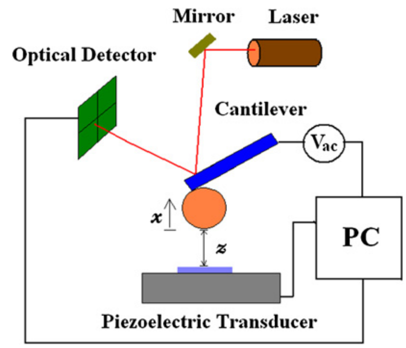

Fig. 1. The schematic of Casimir force actuated system.

mass of the sphere–cantilever ensemble, c is speed of light and $\hbar$ is the reduced Plank constant, z is the separation distance when the sphere is at equilibrium and $r_c(x_1 + z)$ is the conductivity correction to the ideal Casimir force [25]. In this formulation, the separation gap $d = x_1 + z$ is varying with respect to time as the sphere is vibrating. z is the parameter to be estimated. Although the sphere radius R can be measured by SEM in practice, it is assumed to be an unknown parameter in our method. In that case, the estimation of the separation gap is not affected by the measurement error in R. It can be verified that F is a known local Lipschitz continues function of x_1. System (1) is assumed to satisfy the following property:

Property 1. The system states x satisfy:

$$\lim_{t\to\infty} \big(x(t) - x(t - T)\big) = 0 \quad (3)$$

where T is a known constant. This is true in real dynamics so far even if severe nonlinear phenomena like hysteresis, bistability and harmonics are observed or predicted.

In this Letter, our objective is to estimate the unknown parameter z based on the system states x.

3. Observer design

Here, we present an observer to estimate the total force $F(t)$ first. The observer is designed to be:

$$\dot{\hat{x}}_2 = -\mu x_2 - b_1(\hat{x}_2 - x_2) - kx_1 + u + \hat{F}(t) \quad (4)$$

where $b_1 > 0$ and the update law for $\hat{F}(t)$ is chosen to be:

$$\hat{F}(t) = -q(t)\frac{l_\varepsilon \tilde{x}_2(t)}{2} \quad (t < T),$$
$$\hat{F}(t) = \hat{F}(t - T) - \frac{l_\varepsilon \tilde{x}_2(t)}{2} \quad (t \geqslant T) \quad (5)$$

where $\tilde{x}_2 = \hat{x}_2 - x_2$ and $l_\varepsilon > 0$ is the gain to control the speed of convergence. $q(t)$ is a continuous function satisfying $q(0) = 0$ and $q(T) = 1$. Here we choose $q(t) = t/T$. It is easy to verify that with our chosen update law, $\hat{F}(t)$ is continuous everywhere. The error dynamics is governed by the following equation:

$$\dot{\tilde{x}}_2 + b_1\tilde{x}_2 = \hat{F}(t) - F(t). \quad (6)$$

With the proposed observer and the update law, we have the following result:

Theorem 1. *Given the plant as described in (1), with the observer in (4) and the update law in (5), the following statements are true:*