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Coherent polarization driven by external electromagnetic fields

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ABSTRACT

The coherent interaction of the electromagnetic radiation with an ensemble of polarizable, identical particles with two energy levels is investigated in the presence of external electromagnetic fields. The coupled non-linear equations of motion are solved in the stationary regime and in the limit of small coupling constants. It is shown that an external electromagnetic field may induce a macroscopic occupation of both the energy levels of the particles and the corresponding photon states, governed by a long-range order of the quantum phases of the internal motion (polarization) of the particles. A lasing effect is thereby obtained, controlled by the external field. Its main characteristics are estimated for typical atomic matter and atomic nuclei. For atomic matter the effect may be considerable (for usual external fields), while for atomic nuclei the effect is extremely small (practically insignificant), due to the great disparity in the coupling constants. In the absence of the external field, the solution, which is non-analytic in the coupling constant, corresponds to a second-order phase transition (super-radiance), which was previously investigated.

1. Introduction

In a previous paper [1], the coherent interaction of the electromagnetic radiation with an ensemble of polarizable, identical particles with two energy levels has been investigated in the absence of external electromagnetic fields, and the corresponding coupled non-linear equations of motion have been solved. It was shown that the solution has a non-perturbational character (it is non-analytic in the coupling constant). The main role in this problem is played by a dimensionless coupling constant

$$\lambda = \sqrt{\frac{2\pi}{3a^3\hbar\omega_0}}\frac{J_{01}}{\omega_0}, \qquad (1)$$

where J_{01} is the matrix element of the current associated with each particle, a is the mean inter-particle distance and $\hbar\omega_0 = \varepsilon_1 - \varepsilon_0$ is the energy separation between the two levels. It was shown [1] that, at zero temperature, the two levels $\varepsilon_{0,1}$ and the corresponding photon states $\hbar\omega_0$ are macroscopically occupied, provided $\lambda > 1$; at finite temperature, this coherent state sets up for $\lambda > 2$ and below a critical temperature T_c (given by $T_c \simeq \lambda^2\hbar\omega_0/8$). This second-order phase transition is usually known as a super-radiance transition [2–8]; it corresponds to a long-range order of the quantum phases (a lattice of coherence domains) [1], associated with the internal motion (polarization) of the particles.

For numerical estimates we may take $J_{01} = \omega_0 p$, where $p = el$ is the dipole momentum of the particles, l being the distance over which an electron charge e is displaced in the polarization process. For typical atomic matter we may take, for illustrative purposes, $l = a_0 = 0.53$ Å (Bohr radius), $\hbar\omega_0 = 1$ eV and $a = 3$ Å ($p = 2.4 \times 10^{-18}$ esu). We get $\lambda \simeq 0.5$, which is insufficient for setting up the coherent state. Similarly, for atomic nuclei we may take $l = 1$ fm (10^{-13} cm), $\hbar\omega_0 = 1$ MeV and $a = 3$ Å, and get $\lambda \simeq 10^{-8}$, which is an extremely small value for the coupling constant.

We turn our attention in this Letter to the presence of an external electromagnetic field, whose coherent interaction with the ensemble of particles may lead to a lasing effect. We get here the solution of the coupled non-linear equations of motion in the presence of an external field, in the stationary regime and in the limit of small values of the coupling constants. It is shown that the two levels and the corresponding photon states are macroscopically occupied, to an extent which depends on the coupling constant λ and the external field, leading thus to a lasing effect. While for atomic matter ($\lambda \simeq 0.5$) this effect may be considerable (for usual field intensities), it is extremely small (practically insignificant) for atomic nuclei ($\lambda \simeq 10^{-8}$). The problem is similar with the well-known "semi-classical theory" of the laser, which has been extensively investigated, by various approaches and from many angles [9–20]. It is worth noting that the theoretical considerations presented here pertain to a consequent field-theoretical approach to the coherent interaction of matter with electromagnetic radiation, as distinct

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from the usual semi-classical approaches of the current theories of the laser (see, for instance, Refs. [21–23]).

2. Coherent interaction

As it is well known, the electromagnetic field is described by the vector potential

$$\mathbf{A}(\mathbf{r}) = \sum_{\mu\mathbf{k}} \sqrt{\frac{2\pi\hbar c^2}{V\omega_k}} \left[\mathbf{e}_\mu(\mathbf{k}) a_{\mu\mathbf{k}} e^{i\mathbf{kr}} + \mathbf{e}^*_\mu(\mathbf{k}) a^*_{\mu\mathbf{k}} e^{-i\mathbf{kr}}\right] \tag{2}$$

in the standard Fourier representation, with the transverse gauge $\operatorname{div}\mathbf{A} = 0$, where c is the velocity of light, V is the volume, $\omega_k = ck$ is the frequency and $\mathbf{e}_\mu(\mathbf{k})$ are the polarization vectors, $\mathbf{e}_\mu(\mathbf{k})\mathbf{k} = 0$, $\mathbf{e}_\mu(\mathbf{k})\mathbf{e}^*_\mu(\mathbf{k}) = \delta_{\mu\nu}$ $(\mu,\nu = \pm 1)$, $\mathbf{e}_{-\mu}(-\mathbf{k}) = \mathbf{e}^*_\mu(\mathbf{k})$. The electric and magnetic fields are given by $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$ and, respectively, $\mathbf{H} = \operatorname{curl}\mathbf{A}$, and three Maxwell's equations are satisfied: $\operatorname{curl}\mathbf{E} = -\frac{1}{c}\partial\mathbf{H}/\partial t$, $\operatorname{div}\mathbf{H} = 0$, $\operatorname{div}\mathbf{E} = 0$. The time dependence is included in the Fourier coefficients $a_{\mu\mathbf{k}}$, $a^*_{\mu\mathbf{k}}$.

We use a similar expression for the external vector potential $\mathbf{A}^0(\mathbf{r})$, the corresponding Fourier coefficients being denoted by $a^0_{\mu\mathbf{k}}$, $a^{0*}_{\mu\mathbf{k}}$, with a prescribed time-dependence.

We use also the classical lagrangian of the radiation field

$$L_f = \frac{1}{8\pi}\int d\mathbf{r}\left(E^2 - H^2\right), \tag{3}$$

which can be expressed by means of the Fourier coefficients $a_{\mu\mathbf{k}}$, $a^*_{\mu\mathbf{k}}$, and the interaction lagrangian

$$\begin{aligned} L_{int} &= \frac{1}{c}\int d\mathbf{r}\cdot\mathbf{j}(\mathbf{A}+\mathbf{A}_0) \\ &= \sum_{\mu\mathbf{k}}\sqrt{\frac{2\pi\hbar}{\omega_k}}\left[\mathbf{e}_\mu(\mathbf{k})\mathbf{j}^*(\mathbf{k})\left(a_{\mu\mathbf{k}} + a^0_{\mu\mathbf{k}}\right)\right. \\ &\quad \left. + \mathbf{e}^*_\mu(\mathbf{k})\mathbf{j}(\mathbf{k})\left(a^*_{\mu\mathbf{k}} + a^{0*}_{\mu\mathbf{k}}\right)\right], \end{aligned} \tag{4}$$

where $\mathbf{j}(\mathbf{k})$ is the Fourier transform of the current density,

$$\mathbf{j}(\mathbf{r}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}}\mathbf{j}(\mathbf{k})e^{i\mathbf{kr}}, \tag{5}$$

with $\operatorname{div}\mathbf{j} = 0$ (continuity equation). The Euler–Lagrange equations for the lagrangian $L_f + L_{int}$ lead to the wave equation with sources

$$\ddot{a}_{\mu\mathbf{k}} + \ddot{a}^*_{-\mu-\mathbf{k}} + \omega_k^2\left(a_{\mu\mathbf{k}} + a^*_{-\mu-\mathbf{k}}\right) = \sqrt{\frac{8\pi\omega_k}{\hbar}}\mathbf{e}^*_\mu(\mathbf{k})\mathbf{j}(\mathbf{k}), \tag{6}$$

which is the fourth Maxwell's equation $\operatorname{curl}\mathbf{H} = (1/c)\partial\mathbf{E}/\partial t + 4\pi\mathbf{j}/c$.

We consider a set of N independent, non-relativistic, identical particles labelled by $i = 1,\ldots,N$ $(N \gg 1)$ and write the hamiltonian corresponding to their internal degrees of freedom as $H_s = \sum_i H_s(i)$. We introduce a set of orthonormal eigenfunctions $\varphi_n(i)$, where ε_n is the energy level of the n-state, and construct also a set of orthonormal eigenfunctions

$$\psi_n = \frac{1}{\sqrt{N}}\sum_i e^{i\theta_{ni}}\varphi_n(i), \tag{7}$$

where θ_{ni} are some undetermined phases.

The field operator

$$\Psi = \sum_n b_n\psi_n, \tag{8}$$

with boson-like commutation relations $[b_n, b^*_m] = \delta_{nm}$, $[b_n, b_m] = 0$, leads to the (macroscopic) number of particles $N = \sum_n b^*_n b_n$ and to the lagrangian

$$L_s = \frac{1}{2}\sum_n i\hbar\left[b^*_n\dot{b}_n - \dot{b}^*_n b_n\right] - \sum_n \varepsilon_n b^*_n b_n, \tag{9}$$

where $H_s = \sum_n \varepsilon_n b^*_n b_n$ is the hamiltonian of the ensemble of particles. The corresponding equation of motion $i\hbar\dot{b}_n = \varepsilon_n b_n$ is Schrodinger's equation.

The current density associated with this ensemble of particles can be written as

$$\mathbf{j}(\mathbf{r}) = \sum_i \mathbf{J}(i)\delta(\mathbf{r}-\mathbf{r}_i) = \frac{1}{V}\sum_{i\mathbf{k}}\mathbf{J}(i)e^{-i\mathbf{kr}_i}e^{i\mathbf{kr}} = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}}\mathbf{j}(\mathbf{k})e^{i\mathbf{kr}}, \tag{10}$$

where $\mathbf{r}_i$ is the position of the ith particle and $\mathbf{J}(i)$ is the current associated with one particle. Now, making use of Eqs. (8) and (10), it is easy to see that the interaction lagrangian given by Eq. (4) can be written as

$$L_{int} = \sum_{nm\mu\mathbf{k}}\sqrt{\frac{2\pi\hbar}{V\omega_k}}F_{nm}(\mu\mathbf{k})\left(a_{\mu\mathbf{k}} + a^*_{-\mu-\mathbf{k}} + a^0_{\mu\mathbf{k}} + a^{0*}_{-\mu-\mathbf{k}}\right)b^*_n b_m, \tag{11}$$

where

$$F_{nm}(\mu\mathbf{k}) = \frac{1}{N}\sum_i \mathbf{e}_\mu(\mathbf{k})\mathbf{J}_{nm}(i)e^{i\mathbf{kr}_i - i(\theta_{ni}-\theta_{mi})}. \tag{12}$$

$\mathbf{J}_{nm}(i)$ being the matrix element of the current associated with the ith particle.

For any pair (n,m) of levels, the quantum phases θ_{ni} can be arranged in a periodic lattice with the shortest (generating) reciprocal vectors denoted by $\mathbf{k}_r$, $r = 1,2,3$. For a given pair (n,m) we take these vectors as being equal in magnitude, $k_r = k_0$ and $\omega_0 = ck_0$ [1]. Under these circumstances the phase in Eq. (12) may satisfy the condition $\mathbf{k}_r\mathbf{r}_{pi} - (\theta_{ni} - \theta_{mi}) = \text{const}$, where p labels the unit cells of the phase lattice. This condition was called the coherence condition in Ref. [1]. Then, the interaction lagrangian acquires a simple form, which, limiting ourselves to only two levels, and using the coherent states operators [24] $b_{0,1}|\beta_{0,1}\rangle = \beta_{0,1}|\beta_{0,1}\rangle$, can be written as

$$L_{int} = \sqrt{\frac{2\pi\hbar}{V\omega_0}}J_{01}\left(\alpha + \alpha^* + \alpha^0 + \alpha^{0*}\right)\left(\beta^*_1\beta_0 + \beta_1\beta^*_0\right), \tag{13}$$

where we have assumed $J_{00} = J_{11} = 0$. In Eq. (13) we have also replaced the photon operators $a_{\mu\mathbf{k}_r}$, $k_r = k_0$, by c-numbers α, the same for any polarization μ and any direction of the vectors $\mathbf{k}_r$, and similarly for the external field. We note that the external field depends on time; we take $\alpha^0 + \alpha^{0*} = 2|\alpha^0|\cos\omega_0 t$. A similar replacement of the field operators by c-numbers is made in the free lagrangians of the field and particles. The summation over $\mu\mathbf{k}_r$, $k_r = k_0$, in the field lagrangian L_f gives a factor 12, for a three-dimensional lattice (three $\pm\mathbf{k}_r$'s and two polarizations). This factor can be absorbed in the photon operators, so we can write down the full "classical" lagrangian

$$\begin{aligned} L_f &= \frac{\hbar}{4\omega_0}\left(\dot{\alpha}^2 + \dot{\alpha}^{*2} + 2|\dot{\alpha}|^2\right) - \frac{\hbar\omega_0}{4}\left(\alpha^2 + \alpha^{*2} + 2|\alpha|^2\right), \\ L_s &= \frac{1}{2}i\hbar\left(\beta^*_0\dot{\beta}_0 - \dot{\beta}^*_0\beta_0 + \beta^*_1\dot{\beta}_1 - \dot{\beta}^*_1\beta_1\right) - \left(\varepsilon_0|\beta_0|^2 + \varepsilon_1|\beta_1|^2\right), \\ L_{int} &= \frac{g}{\sqrt{N}}\left[\alpha + \alpha^* + \alpha^0 + \alpha^{0*}\right]\left(\beta_0\beta^*_1 + \beta_1\beta^*_0\right), \end{aligned} \tag{14}$$