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Geodesic acoustic modes and zonal flows in toroidally rotating tokamak plasmas

V.P. Lakhin [a], V.I. Ilgisonis [a], A.I. Smolyakov [a,b,*]

[a] RRC "Kurchatov Institute", 123182, Moscow, Russian Federation
[b] University of Saskatchewan, 116 Science Place, Saskatoon, S7N 5E2, Canada

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ABSTRACT

The effect of equilibrium toroidal rotation on the rotational eigen-modes in large aspect ratio tokamak is studied. The case of equilibrium with uniform plasma density on magnetic surfaces is considered. It is shown that the toroidal rotation results in a frequency up-shift of ordinary Geodesic Acoustic Modes. A new unstable low frequency branch of the continuum modes is found. This mode appears as a consequence of the non-uniform plasma pressure created by the centrifugal force on the magnetic surfaces. This mode represents a linear eigen-mode counterpart of Zonal Flow modes. It is shown that the growth rate of such a mode increases with the increase of the angular velocity of toroidal rotation.

Fluctuations of $m = n = 0$ component of the electrostatic potential in a tokamak lead to the perturbations of poloidal and toroidal plasma rotation. Such rotational perturbations spontaneously occur in the form of zonal flows (ZFs) and geodesic acoustic modes (GAMs) and have been observed experimentally in many tokamaks [1–3]. Such modes are considered to play an important role in regulating the anomalous, turbulent transport in modern tokamaks (see [4] and references therein) either due to energy sink from turbulence to rotational modes and/or due to the shearing effect of the rotation on the turbulent transport. In a static toroidal plasmas, geodesic acoustic modes represent the linear eigen-modes with the frequency of the order of $\omega \sim c_s/R_0$, where c_s is the sound velocity and R_0 is the major radius of the magnetic axis. These modes are linearly stable in the framework of ideal MHD. On the contrary, the low frequency zonal flows have no linear counterpart and are driven non-linearly due to energy transfer from turbulence.

Plasmas in present day tokamak experiments often have finite rotation velocity. In experiments with an unbalanced tangential neutral beam injection the equilibrium toroidal velocities may approach the sound velocities. Therefore, it is interesting and important to study the influence of toroidal plasma rotation on ZFs and GAMs. It is shown in this Letter that toroidal rotation, in addition to the modification of the standard GAM modes existing in the static case of non-rotating plasma, leads to the appearance of a new low frequency mode which can be classified as a linear eigenmode of the zonal flow.

In the previous studies the effect of toroidal rotation on ZFs and GAMs has been investigated within an MHD description of plasma [5,6]. The new GAM branch induced by toroidal rotation has been found for a particular equilibrium in which the plasma temperature is assumed constant on the magnetic flux surfaces. The frequency of the new GAM is always lower than the frequency of ordinary GAM [7] and tends to zero when the angular velocity of toroidal rotation tends to zero. The effect of toroidal rotation on the Alfvén continua, which are the GAM modification to the case of finite poloidal and toroidal wave numbers, has been also studied in [8,9] in the frame of MHD approach. Different plasma equilibria have been considered. In Ref. [8] the equilibrium with isothermal magnetic surfaces has been taken, and in Ref. [9] – the equilibrium with uniform plasma density on the magnetic surfaces. In the latter Letter the existence of unstable Alfvén continua has been noted. Also, the effect of equilibrium mass flow parallel to the magnetic field on the Alfvén continua has been studied in Ref. [10] by the kinetic approach.

Approximation of the constant temperature on the magnetic flux surface is based on the assumption of high thermal conductivity along magnetic field lines. However, the parallel heat conductivity can be not too effective in low aspect ratio tokamaks with large fraction of trapped particles or in tokamaks with intensive auxiliary heating, where the power absorption profiles can be rather complicated (see, e.g., [11,12]). As a result, the other types of equilibria of flowing plasmas with temperature, which is not constant along the magnetic flux surfaces, are also possible. In particular, the equilibria, in which either the plasma mass density, ρ_0, or the plasma entropy, p_0/ρ_0^{Γ} (p_0 is the equilibrium pressure, Γ is the adiabatic index), are constant on the magnetic flux surface, are also known (see, e.g., [13] and references therein). In this Letter we will consider the properties of GAMs and ZFs in the case of equilibrium, in which the mass density is uniform on the flux surfaces and will show that they are qualitatively different from the case of equilibrium with isothermal flux surfaces. Another equilib-

* Corresponding author at: University of Saskatchewan, 116 Science Place, Saskatoon, S7N 5E2, Canada.
E-mail address: andrei.smolyakov@usask.ca (A.I. Smolyakov).

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rium – with isentropic flux surfaces – is considered as a particular case in Ref. [14], where GAMs are studied for equilibrium with an arbitrary rotation (toroidal and poloidal).

We assume the equilibrium with toroidal rotation and describe the magnetic field of axisymmetric tokamak and the toroidal plasma flow by the expressions

$$\mathbf{B}_0 = I(\psi)\nabla\varphi + \nabla\psi \times \nabla\varphi, \qquad \mathbf{v}_0 = R^2\Omega(\psi)\nabla\varphi \tag{1}$$

where ψ is the magnetic flux, φ is the toroidal angle, R is the major radius of tokamak, and I is the poloidal current stream function.

If the equilibrium plasma mass density is assumed constant on the magnetic flux surfaces, $\rho_0 = \rho_0(\psi)$, the centrifugal force creates a non-uniform equilibrium plasma pressure, which is given by

$$p_0 = P(\psi) + \frac{1}{2}\rho_0\Omega^2R^2. \tag{2}$$

We consider the electrostatic, axisymmetric ($\partial/\partial\varphi = 0$) perturbations of the above equilibrium with the toroidal mass flow and take their spatio-temporal dependence in the form $f' = f'(\psi,\theta)\exp(-i\omega t)$. Here ω is the frequency of the perturbation, and $f'(\psi,\theta)$ is a periodic function of poloidal angle θ, $f'(\psi,\theta + 2\pi) = f'(\psi,\theta)$. In the framework of one-fluid MHD model in the electrostatic approximation [15] such perturbations are described by the linearized set of equations

$$\rho_0\big[-i\omega\mathbf{v}' + (\mathbf{v}_0\cdot\nabla)\mathbf{v}' + (\mathbf{v}'\cdot\nabla)\mathbf{v}_0\big] + \rho'(\mathbf{v}_0\cdot\nabla)\mathbf{v}_0 = -\nabla p' + \frac{1}{c}\mathbf{j}'\times\mathbf{B}_0, \tag{3}$$

$$\mathbf{v}'\times\mathbf{B}_0 = c\nabla\phi', \tag{4}$$

$$-i\omega\rho' + \nabla\cdot(\rho_0\mathbf{v}') = 0, \tag{5}$$

$$-i\omega(p' - c_s^2\rho') + \rho_0^\Gamma\mathbf{v}'\cdot\nabla\left(\frac{p_0}{\rho_0^\Gamma}\right) = 0, \tag{6}$$

$$\nabla\cdot\mathbf{j}' = 0. \tag{7}$$

Here ρ' is the perturbation of mass density, $\mathbf{v}'$ is the perturbed plasma velocity, p' is the perturbation of pressure, $\mathbf{j}'$ is the perturbed current density, and ϕ' is the perturbed electrostatic potential. Eq. (3) is the perturbed equation of the motion, Eq. (4) corresponds to the perturbed Ohm's law in electrostatic limit, Eqs. (5) and (6) are the linearized continuity equation and the linearized adiabate equation, and Eq. (7) is the quasineutrality condition.

It follows from Eq. (4) that $\phi' = \phi'(\psi)$, and therefore the perturbation of plasma velocity lies on the magnetic surfaces of tokamak ($\mathbf{v}'\cdot\nabla\psi = 0$) and takes the form

$$\mathbf{v}' = \frac{c}{B_0^2}\mathbf{B}_0\times\nabla\phi' + \frac{v'_\parallel}{B_0}\mathbf{B}_0 \equiv \frac{1}{B_0^2}\left(v'_\parallel B_0 - cI\frac{d\phi'}{d\psi}\right)\mathbf{B}_0 + cR^2\frac{d\phi'}{d\psi}\nabla\varphi, \tag{8}$$

where $v'_\parallel$ is the perturbation of velocity along the magnetic field. We substitute Eq. (8) in Eqs. (5) and (6) and obtain

$$-i\omega\frac{\rho'}{\rho_0} + \mathbf{B}_0\nabla\cdot\left[\frac{1}{B_0^2}\left(V' - cI\frac{d\phi'}{d\psi}\right)\right] = 0, \tag{9}$$

$$-i\omega(p' - c_s^2\rho') + \frac{1}{B_0^2}\left(V' - cI\frac{d\phi'}{d\psi}\right)\mathbf{B}_0\cdot\nabla p_0 = 0. \tag{10}$$

Hereafter $V' \equiv v'_\parallel B_0$. The component of the equation of the motion (3) along the magnetic field yields

$$-i\omega V' - c\Omega\frac{d\phi'}{d\psi}\mathbf{B}_0\cdot\nabla R^2 - \frac{\rho'\Omega^2}{2\rho_0}\mathbf{B}_0\cdot\nabla R^2 = -\frac{1}{\rho_0}\mathbf{B}_0\cdot\nabla p'. \tag{11}$$

To close the set of equations we follow Ref. [14]. We calculate $\mathbf{j}'$ from Eq. (3), substitute it in Eq. (7) and average the resulting equation along the magnetic field line. As a result, we arrive at the following equation

$$\frac{d}{d\psi}\left\{\int_0^{2\pi}\frac{d\theta}{J}\left(-i\omega\frac{c\rho_0|\nabla\psi|^2}{B_0^2}\frac{d\phi'}{d\psi} + p'I\mathbf{B}_0\cdot\nabla\left(\frac{1}{B_0^2}\right) + \frac{\rho_0\Omega}{B_0^2}V'\mathbf{B}_0\cdot\nabla R^2 + \frac{\rho' I\Omega^2}{2B_0^2}\mathbf{B}_0\cdot\nabla R^2\right)\right\} = 0. \tag{12}$$

We use the straight field line coordinates, and the Jacobian J and the tokamak safety factor q are defined by the expressions

$$J \equiv (\nabla\psi\times\nabla\varphi)\cdot\nabla\theta, \qquad q(\psi) \equiv \frac{I}{JR^2}. \tag{13}$$

Further we restrict ourselves to the case of large aspect ratio tokamaks $R_0/a \equiv 1/\epsilon \gg 1$ (a is the minor radius of the torus) and of low beta plasma $\beta \equiv 8\pi p_0/B_0^2 \sim \epsilon^2$. Furthemore, we assume that the toroidal velocity is sufficiently small, so that $\Omega R \leqslant c_s$. Under such an assumption the effect of plasma rotation does not exceed the plasma pressure effects. To the main order in small parameter ϵ the Shafranov shift can be neglected. The magnetic surfaces of tokamak are considered to be circular and concentric. They can be described by the following expressions:

$$\psi = \psi(r): \quad R \approx R_0 + r\cos\theta, \tag{14}$$

where r is the label of magnetic surface meaning its radius. To the main order in ϵ the functions $f = (p_0, J)$ are constant on the magnetic surfaces. They can be represented in the form

$$f = \bar{f}(\psi) + \tilde{f}(\psi,\theta), \qquad \bar{f}(\psi) = \frac{1}{2\pi}\int_0^{2\pi} f\,d\theta, \tag{15}$$

and $\tilde{f} \sim \epsilon\bar{f}(\psi)$.

Making use of the large-aspect-ratio expansion and taking into account Eqs. (2) and (14), we reduce Eqs. (9)–(12) to

$$-i\omega\frac{\rho'}{\rho_0} + \frac{1}{qR_0\bar{B}_0}\frac{\partial V'}{\partial\theta} + \frac{2cr}{qR_0}\frac{d\phi'}{d\psi}\sin\theta = 0, \tag{16}$$

$$-i\omega\frac{V'}{\bar{B}_0} + \frac{2cr}{q}\Omega\frac{d\phi'}{d\psi}\sin\theta + \frac{1}{qR_0\bar{\rho}_0}\frac{\partial p'}{\partial\theta} = 0, \tag{17}$$

$$-i\omega(p' - \bar{c}_s^2\rho') + \frac{cr\rho_0}{qR_0}M^2\bar{c}_s^2\frac{d\phi'}{d\psi}\sin\theta = 0, \tag{18}$$

$$\frac{d}{d\psi}\left\{\frac{r\rho_0}{\bar{B}_0}\left[i\omega\frac{cr}{q}\frac{d\phi'}{d\psi} + \frac{1}{\pi R_0\rho_0}\int_0^{2\pi}d\theta\, p'\sin\theta + \frac{\Omega}{\pi}\int_0^{2\pi}d\theta\frac{V'}{\bar{B}_0}\sin\theta + \frac{\Omega^2R_0}{2\pi}\int_0^{2\pi}d\theta\frac{\rho'}{\rho_0}\sin\theta\right]\right\} = 0, \tag{19}$$

where $\bar{c}_s^2 = \Gamma\bar{p}_0/\rho_0$ and M is the sonic Mach number, $M^2 = \rho_0\Omega^2R_0^2/\bar{c}_s^2$.

Solving Eqs. (16)–(18) and substituting the perturbations of the mass density, of the pressure and of the parallel velocity in Eq. (19), we finally arrive at the following equation