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Renormalized dynamics in charge qubit measurements by a single electron transistor

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ABSTRACT

We investigate charge qubit measurements using a single electron transistor, with focus on the backaction-induced renormalization of qubit parameters. It is revealed the renormalized dynamics leads to a number of intriguing features in the detector's noise spectra, and therefore needs to be accounted for to properly understand the measurement result. Noticeably, the level renormalization gives rise to a strongly enhanced signal-to-noise ratio, which can even exceed the universal upper bound imposed quantum mechanically on linear-response detectors.

1. Introduction

The issue of measurement lies at the heart of the interpretation of quantum mechanics. The recent upsurge in the interest to the quantum computation has attracted renewed attention to the problem of quantum measurement [1,2]. Various schemes have been proposed for fast readout of a two-level quantum state (qubit). Among them, especially interesting are electrometers whose conductance depends on the charge states of a nearby qubit, such as quantum point contacts (QPC) [3–11] and single electron transistors (SET) [12–20]. It has been shown that the SET detector is better than QPC in many respects [21], and has already been used for quantum measurements [22].

So far, theoretical description of the SET detector has been mainly focused on the backaction-induced dephasing and relaxation, which, from the perspective of information, are consequences of information acquisition by measurement [16–18]. Actually, there is another important backaction which renormalizes the internal structure of the qubit [11], and is often disregarded in the literature. However, this renormalization effect is of essential importance, since it can crucially influence the dynamical process of quantum measurement. It is, therefore, required to have this feature being properly accounted for in order to correctly understand and analyze the measurement results.

In this context, we examine the renormalized dynamics of qubit measurements using an SET detector. The intriguing dynamics arising from the renormalization is manifested unambiguously in the noise spectral of the detector output. It is demonstrated that in the low bias regime, the noise peak reflecting qubit oscillations shifts markedly with the measurement voltages. Furthermore, a peak at zero-frequency arises, as the level renormalization results in a so-called quantum Zeno effect. The output noise spectral allows us to evaluate the "signal-to-noise" ratio, which provides the measurement of detector effectiveness. Noticeably, it is revealed that for the SET detector, the level renormalization leads to a considerably enhanced effectiveness, which can even exceed the upper bound imposed on any linear-response detectors [23].

The Letter is structured as follows. The measurement setup and model Hamiltonian are introduced in the next Section. We sketch the quantum master equation approach in Section 3. The results and discussions are presented in Section 4, which is then followed by the summary in Section 5.

2. Model description

The setup for the measurement of a charge qubit (an electron in a pair of coupled quantum dots) by a single electron transistor is schematically shown in Fig. 1. The entire system Hamiltonian reads $H = H_S + H_B + H'$. The first component

$$H_S = \frac{1}{2}\epsilon\,\sigma_z + \Omega\,\sigma_x + \big(E + \tilde{E}|\alpha\rangle\langle\alpha|\big)\,d^\dagger d \tag{1}$$

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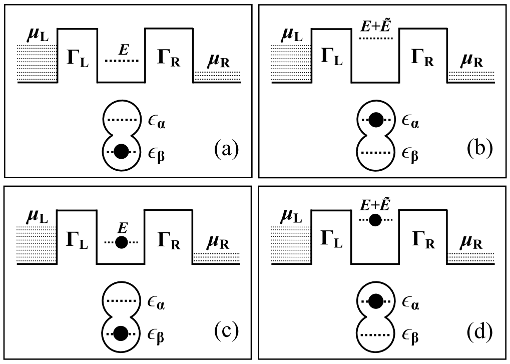

Fig. 1. Schematic setup for a solid-state charge qubit measurements by an SET detector. Possible electron configurations of the measured qubit and SET dot are shown in (a)–(d), respectively.

models the qubit, SET dot, and their coupling, with pseudo-spin operators $\sigma_z \equiv |\alpha\rangle\langle\alpha| - |\beta\rangle\langle\beta|$ and $\sigma_x \equiv |\alpha\rangle\langle\beta| + |\beta\rangle\langle\alpha|$. For the qubit, it is assumed that each dot has only one bound state, i.e., the logic states $|\alpha\rangle$ and $|\beta\rangle$, with level detuning ϵ and interdot coupling Ω. The SET works in the strong Coulomb blockade regime, and only one level is involved in the transport. Here, d ($d^\dagger$) is the annihilation (creation) operator for an electron in the SET dot. The single level (or equivalently, the transport current) depends explicitly on the qubit state, as shown in Fig. 1. It is right this mechanism that makes it possible to acquire the qubit-state information from the SET output.

The second component $H_B = \sum_{\ell=L,R}\sum_k \varepsilon_{\ell k} c^\dagger_{\ell k} c_{\ell k}$ depicts the left and right SET electrodes. Here $c_{\ell k}$ ($c^\dagger_{\ell k}$) denotes the annihilation (creation) operator for an electron in the electrode $\ell \in \{L, R\}$. The electron reservoirs are characterized by the Fermi distribution $f_{L/R}(\omega)$. We set $\mu_L^{eq} = \mu_R^{eq} = 0$ for the equilibrium chemical potentials. An applied measurement voltage V is modeled by different chemical potentials in the left and right electrodes $\mu_{L/R} = \pm V/2$.

Electrons tunneling between SET dot and electrodes is described by the last component $H' = \sum_{\ell k}(t_{\ell k} c^\dagger_{\ell k} d + \text{h.c.}) \equiv \sum_\ell (F^\dagger_\ell d + \text{h.c.})$, where F_ℓ and $F^\dagger_\ell$ are defined implicitly. The tunnel coupling strength between lead ℓ and the SET dot is characterized by $\Gamma_\ell(\omega) = 2\pi\sum_k |t_{\ell k}|^2 \delta(\omega - \varepsilon_{\ell k})$. In what follows, we assume wide bands in the electrodes, which yields energy independent couplings $\Gamma_{L/R}$. Throughout this work, we set $\hbar = e = 1$ for the Planck constant and electron charge, unless stated otherwise.

3. Formalism

3.1. Conditional master equation

To achieve a description of the output from the SET detector, we employ the transport particle-number-resolved reduced density matrices $\rho^{(n_L,n_R)}$, where $n_{L(R)}$ denotes the number of electrons tunneled through the left (right) junction. The corresponding *conditional* quantum master equation reads [11,24–27]

$$\begin{aligned}\dot\rho^{(n_L,n_R)} = &-i\mathcal{L}\rho^{(n_L,n_R)} - \frac{1}{2}\Big\{\big[d^\dagger A^{(-)}\rho^{(n_L,n_R)} + \rho^{(n_L,n_R)}A^{(+)}d^\dagger\big] \\ &- \big[A_L^{(-)}\rho^{(n_L-1,n_R)}d^\dagger + d^\dagger\rho^{(n_L+1,n_R)}A_L^{(+)} \\ &+ A_R^{(-)}\rho^{(n_L,n_R-1)}d^\dagger + d^\dagger\rho^{(n_L,n_R+1)}A_R^{(+)}\big] + \text{h.c.}\Big\},\end{aligned} \tag{2}$$

where $\mathcal{L}$ is defined as $\mathcal{L}(\cdots) \equiv [H_S, (\cdots)]$, and $A^{(\pm)} = \sum_\ell A_\ell^{(\pm)}$, with $A_\ell^{(\pm)} \equiv [C_\ell^{(\pm)}(\pm\mathcal{L}) + iD_\ell^{(\pm)}(\pm\mathcal{L})]d$. Here $C_\ell^{(\pm)}(\pm\mathcal{L}) = \int_{-\infty}^{\infty} dt\, C_\ell^{(\pm)}(t)e^{\pm i\mathcal{L}t}$ are spectral functions. The involving bath correlation functions are respectively $C_\ell^{(+)}(t) = \langle F_\ell^\dagger(t)F_\ell\rangle_B$, and $C_\ell^{(-)}(t) = \langle F_\ell(t)F_\ell^\dagger\rangle_B$, with $\langle\cdots\rangle_B \equiv \text{Tr}_B[(\cdots)\rho_B]$, and ρ_B the local thermal equilibrium state of the SET leads. The involved dispersion functions $D_\ell^{(\pm)}(\pm\mathcal{L})$ can be evaluated via the Kramers–Kronig relation

$$D_\ell^{(\pm)}(\pm\mathcal{L}) = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} d\omega \frac{C_\ell^{(\pm)}(\pm\omega)}{\mathcal{L}-\omega}, \tag{3}$$

where $\mathcal{P}$ denotes the principal value. Physically, the dispersion is responsible for the renormalization [11,28–31].

3.2. Output current

With the knowledge of the above conditional state, the joint probability function for n_L electrons passed through left junction and n_R electrons passed through right junction is determined as $P(n_L, n_R) = \text{Tr}\,\rho^{(n_L,n_R)}$, where $\text{Tr}(\cdots)$ denotes the trace over the system degrees of freedom. The current through junction $\ell \in \{L, R\}$ then reads $I_\ell = \frac{d}{dt}\sum_{n_L,n_R} n_\ell P(n_L, n_R) = \text{Tr}\,\dot N_\ell$, where $N_\ell \equiv \sum_{n_L,n_R} n_\ell P(n_L, n_R)$ can be calculated via its equation of motion

$$\frac{d}{dt}N_\ell = -i\mathcal{L}N_\ell - \mathcal{R}N_\ell + \mathcal{T}_\ell^{(-)}\rho, \tag{4a}$$

with

$$\mathcal{R}(\cdots) = -\frac{1}{2}\big[d^\dagger, A^{(-)}(\cdots) - (\cdots)A^{(+)}\big] + \text{h.c.}, \tag{4b}$$

$$\mathcal{T}_\ell^{(\pm)}(\cdots) = \frac{1}{2}\big[A_\ell^{(-)}(\cdots)d^\dagger \pm d^\dagger(\cdots)A_\ell^{(+)}\big] + \text{h.c.} \tag{4c}$$

Straightforwardly, the transport current through junction ℓ is $I_\ell(t) = \text{Tr}[\mathcal{T}_\ell^{(-)}\rho(t)]$. Here $\rho(t)$ is the *unconditional* density matrix, which simply satisfies

$$\dot\rho = -i\mathcal{L}\rho - \mathcal{R}\rho. \tag{5}$$

3.3. Current noise spectrum

In continuous weak measurement of qubit oscillations, the most important output is the spectral density of the current. The circuit current of the SET detector, according to the Ramo–Shockley theorem [32], is $I(t) = \eta_L I_L + \eta_R I_R$, where the coefficients η_L and η_R depend on junction capacitances and satisfy $\eta_L + \eta_R = 1$. Together with the charge conservation law $I_L = I_R + \dot Q$, where Q represents the electron charge in the SET dot, one readily obtains $I(t)I(0) = \eta_L I_L(t)I_L(0) + \eta_R I_R(t)I_R(0) - \eta_L\eta_R\dot Q(t)\dot Q(0)$. Accordingly, the circuit noise spectral is a sum of three parts [18,25,33,34]

$$S(\omega) = \eta_L S_L(\omega) + \eta_R S_R(\omega) - \eta_L\eta_R S_{ch}(\omega), \tag{6}$$

with $S_{L(R)}(\omega)$ the noise spectral of the left (right) junction current, and $S_{ch}(\omega)$ the charge fluctuations in the SET dot. The noise spectral of tunneling current $S_{L/R}$ can be evaluated via the MacDonald's formula [35–37]

$$S_\ell(\omega) = 2\omega\int_0^\infty dt\,\sin(\omega t)\frac{d}{dt}\big[\langle n_\ell^2(t)\rangle - (\bar I t)^2\big], \tag{7}$$

where $\bar I \equiv I(t\to\infty)$ is the stationary current, and $\langle n_\ell^2(t)\rangle \equiv \sum_{n_L,n_R} n_\ell^2 P(n_L, n_R)$. With the help of the conditional master equation (2), it can be shown that

$$\frac{d}{dt}\langle n_\ell^2(t)\rangle = \text{Tr}\big[2\mathcal{T}_\ell^{(-)}N_\ell(t) + \mathcal{T}_\ell^{(+)}\rho_{st}\big]. \tag{8}$$

Here $N_\ell(t)$ can be found from Eq. (4a), and ρ_{st} is the stationary solution of the unconditional master equation (5).