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Approximate rational Jacobi elliptic function solutions of the fractional differential equations via the enhanced Adomian decomposition method ☆

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ABSTRACT

In this Letter, an enhanced Adomian decomposition method which introduces the h-curve of the homotopy analysis method into the standard Adomian decomposition method is proposed. Some examples prove that this method can derive successfully approximate rational Jacobi elliptic function solutions of the fractional differential equations.

1. Introduction

The approximate solutions of the fractional differential equations have become more and more attractive in recent years, which is attributed mainly to the fact that the fractional differential equations can describe many important phenomena in electromagnetic, acoustics, viscoelasticity, electrochemistry, cosmology and material science [1–4]. With the development of symbolic computer softwares which allow us to perform some complicated and tedious differential calculation on a computer, the approximate solutions of the fractional differential equations have been found increasingly. Some analytical methods which are used traditionally to give the approximate and exact solutions of the linear and nonlinear differential equations of the integer order derivatives, like the Adomian decomposition method (ADM) [5,6], the homotopy analysis method (HAM) [7–11], the variational iteration method [12], the homotopy perturbation method [13], are generalized to solve various linear and nonlinear fractional equations [14–22].

In the approaches mentioned above, it is acknowledged that the ADM and the HAM are straightforward and effective algebraic algorithms to obtain approximate solutions for lots of nonlinear integer order and fractional differential equations. They are relatively new techniques and independent of whether or not there exist small parameters in the considered equation. This two methods can overcome the foregoing restrictions and limitations of perturbation techniques so that they provide us with a possibility to analyze strongly nonlinear problems. The ADM was firstly proposed by the American mathematician, G. Adomian [5,6]. The crux of the ADM is to decompose a solution of the equation into an infinite series and substitute the series with computable Adomian polynomials for the nonlinear term. The HAM is raised by Liao [7–11] based on the homotopy of topology. Different from the general perturbation techniques, the so-called deformation equations constructed by Liao include an especial parameter h that provides us with a convenient way to adjust and control convergence region. Some empirical observations [7–11,23,24] have indicated that choosing a proper h can improve the accuracy of the approximate solutions. The aim of the present Letter to introduce the h-curve of HAM into the standard ADM and propose an enhanced ADM. In this new framework, the approximate solutions are in terms of the infinite series composed of computable components with a new parameter h. We can follow the theory of Liao's h-curve to identify optimally the value of h, and then constructed the approximate solutions with higher precision. some examples of fractional derivatives are illustrated to show the advantages of the proposed method for obtaining approximate rational Jacobi elliptic function solutions of the fractional differential equations with initial conditions.

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The Letter has been organized as follows. In Section 2, the standard ADM and the enhanced ADM are described succinctly. In Section 3, some examples of fractional time-derivatives are illustrated to prove the advantage of the enhanced ADM than the standard one. Conclusions are presented in Section 4.

2. Description of the methods

In what follows, the main procedures of the standard and enhanced ADMs that will be used in this Letter are outlined.

Consider the general nonlinear differential equation with a physical field $u(x,t)$ in two variables x, t,

$$[\mathcal{L}+\mathcal{R}+\mathcal{N}]u(x,t)=g(x,t), \tag{1}$$

which subjects to the initial condition

$$u(x,0)=f(x), \tag{2}$$

where $\mathcal{L}$ is an easily or trivially invertible linear operator, $\mathcal{R}$ is the remaining linear part, $\mathcal{N}$ is a nonlinear operator, and $g(x,t)$ is a known analytical function.

Eq. (1) can be rewritten as

$$\mathcal{L}u=g-[\mathcal{R}+\mathcal{N}]u. \tag{3}$$

Because $\mathcal{L}$ is invertible, Eq. (1) is equivalent to

$$u=\Phi+\mathcal{L}^{-1}g-\mathcal{L}^{-1}[\mathcal{R}+\mathcal{N}]u, \tag{4}$$

where Φ is determined by its initial value problems.

The standard ADM [5,6] suggests the solution u can be decomposed into the infinite series with components

$$u=\sum_{n=0}^{\infty}u_n, \tag{5}$$

and the nonlinear term $\mathcal{N}u$ is decomposed, as follows:

$$\mathcal{N}u=\sum_{n=0}^{\infty}A_n, \tag{6}$$

where A_n are Adomian polynomials, defined by

$$A_n=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\mathcal{N}\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \quad n\geqslant 0. \tag{7}$$

Substituting the decomposition series (5) and (6) into both sides of (4) gives the following relationship

$$\sum_{n=0}^{\infty}u_n=\Phi+\mathcal{L}^{-1}g-\mathcal{L}^{-1}\mathcal{R}\sum_{n=0}^{\infty}u_n-\mathcal{L}^{-1}\sum_{n=0}^{\infty}A_n. \tag{8}$$

From the above equation, the standard ADM defines the components u_n by the following recursive relationship

$$u_0=\Phi+\mathcal{L}^{-1}g, \qquad u_{n+1}=-\mathcal{L}^{-1}[\mathcal{R}u_n+A_n]. \tag{9}$$

In the computation process, the standard ADM includes actually a parameter λ. Based on the idea of HAM, we insert another parameter h into Eq. (9) and then construct new components, defined by

$$u_0=\Phi+\mathcal{L}^{-1}g, \tag{10a}$$

$$u_{k+1}=-\mathcal{L}^{-1}[\mathcal{R}u_k+A_k], \tag{10b}$$

$$\widetilde{u}_n=h\sum_{k=0}^{n-1}C_{n-1}^{k}(1-h)^{n-1-k}h^k u_{k+1}, \quad n\geqslant 1. \tag{10c}$$

In this new recursive mode, a solution u of Eq. (1) is a infinite series with the computable components $\widetilde{u}_n$ which contains a new parameter h, namely

$$u=\sum_{n=0}^{\infty}\widetilde{u}_n. \tag{11}$$

Eq. (10a) is linear and thus can be easily solved using the initial conditions, all components of series $\{u_n\}$ $(n=1,2,\ldots)$ are determinable by using Eq. (10b). In line with Eq. (10c), we can obtain the expressions of $\{\widetilde{u}_n\}$ $(n=1,2,\ldots)$. Substituting $u_0,\widetilde{u}_1,\widetilde{u}_2,\ldots,\widetilde{u}_n,\ldots$ in Eq. (11) and using Liao' h-curve can derive approximate solutions with higher accuracy.