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Towards Google matrix of brain

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ABSTRACT

We apply the approach of the Google matrix, used in computer science and World Wide Web, to description of properties of neuronal networks. The Google matrix **G** is constructed on the basis of neuronal network of a brain model discussed in PNAS 105 (2008) 3593. We show that the spectrum of eigenvalues of **G** has a gapless structure with long living relaxation modes. The PageRank of the network becomes delocalized for certain values of the Google damping factor α. The properties of other eigenstates are also analyzed. We discuss further parallels and similarities between the World Wide Web and neuronal networks.

1. Introduction

More than 50 years ago John von Neumann traced first parallels between architecture of the computer and the brain [1]. Since that time computers became an unavoidable element of the modern society forming a computer network connected by the World Wide Web (WWW). The WWW demonstrates a continuous growth approaching to 10^{11} web pages spread all over the world (see e.g. http://www.worldwidewebsize.com/). This number starts to become even larger than 10^{10} neurons in the brain. Each neuron can be viewed as an independent processing unit connected with about 10^4 other neurons by synaptic links (see e.g. [2–4]). About 20% of these links are unidirectional [5] and hence the brain can be viewed as a directed network of neuron links. At present, more and more experimental information about neurons and their links becomes available and the investigation of properties of neuronal networks attracts an active interest of many groups (see e.g. [6–13]).

The WWW is also a directed network where a site j points to a site i but no necessary vice versa. The classification of web sites and information retrieval from such an enormous data base as the WWW becomes a formidable challenge of modern society where the search engines like Google are used by Internet users in everyday life. An efficient way to classify and extract the information from WWW is based on the PageRank Algorithm (PRA), proposed by Brin and Page in 1998 [14], which forms the basis of the Google search engine. The PRA is based on the construction of the Google matrix which can be written as (see e.g. [15] for details):

$$\mathbf{G} = \alpha \mathbf{S} + (1-\alpha)\mathbf{E}/N. \quad (1)$$

Here the matrix **S** is constructed from the adjacency matrix **A** of directed network links between N nodes so that $S_{ij} = A_{ij}/\sum_k A_{kj}$ and the elements of columns with only zero elements are replaced by $1/N$. The second term in r.h.s. of (1) describes a finite probability $1-\alpha$ for WWW surfer to jump at random to any node so that the matrix elements $E_{ij} = 1$. This term with the Google damping factor α stabilizes the convergence of PRA introducing a gap between the maximal eigenvalue $\lambda = 1$ and other eigenvalues λ_i. As a result the first eigenvalue has $\lambda_1 = 1$ and the second one has $|\lambda_2| \leqslant \alpha$. Usually the Google search uses the value $\alpha = 0.85$ [15]. By the construction $\sum_i G_{ij} = 1$ so that the asymmetric matrix **G** belongs to the class of Perron–Frobenium operators which naturally appear in the ergodic theory [16] and dynamical systems with Hamiltonian or dissipative dynamics [17]. From the view point of dynamical systems the matrix **G** describes time evolution in presence of noise and coarse-graining. In fact, dynamical systems naturally generate so-called Ulam networks [18,19] where the nodes are formed by coarse-gaining cells in the phase space and **G** gives a time evolution of probability in a coarse-grained phase space.

The right eigenvector at $\lambda = 1$ is the PageRank vector with positive elements p_j and $\sum_j p_j = 1$. The classification of nodes in the decreasing order of p_j values is used to classify importance of WWW nodes as it is described in more detail in [15]. The PageRank can be efficiently obtained by a multiplication of a ran-

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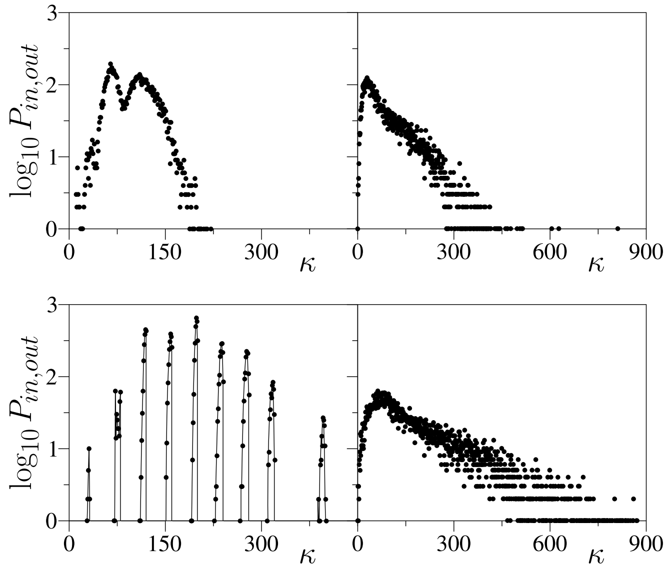

Fig. 1. Distribution of *ingoing* (left panels) and *outgoing* (right panels) links κ: P_{in} and P_{out} give number of nodes with κ ingoing and *outgoing* links respectively. Top panels: unweighted links; bottom panels: weighted links.

dom vector by **G** which is of low cost since in average there are only about ten nonzero elements in a typical line of **G** of WWW. This procedure converges rapidly to the PageRank.

Fundamental investigations of the PageRank properties of the WWW have been performed in the computer science community (see e.g. [20–25]; involvement of physicists is visible, e.g. [26], but less pronounced). It was established that the PageRank is satisfactory characterized by an algebraic decay $p_j \sim 1/j^{\beta}$ with j being the ordering index and $\beta \approx 0.9$; the number of nodes with the PageRank p scales as $N_n \sim 1/p^{\nu}$ with the numerical value of the exponent $\nu = 1 + 1/\beta \approx 2.1$ [15,20]. It is known that such type of algebraic dependencies appear in various types of scale-free networks [27]. The PageRank classification finds its applications not only for the WWW but also for the network of article citations in Physical Review as it is described in [28,29]. This shows that the approach based on the Google matrix can be applied to vary different types of networks.

In this work we construct the Google matrix **G** for a model of brain analyzed in [11]. The properties of the spectrum and the eigenstates of **G** are described in the next Section 2. The results are discussed in Section 3.

2. Numerical results

In our studies we use the brain model of [11] which is a large-scale thalamocortical model based on experimental measures in several mammalian species. The model spans three anatomical scales. (i) It is based on global (white-matter) thalamocortical anatomy obtained by means of diffusion tensor imaging (DTI) of a human brain. (ii) It includes multiple thalamic nuclei and six-layered cortical microcircuitry based on in vitro labeling and three-dimensional reconstruction of single neurons of cat visual cortex. (iii) It has 22 basic types of neurons with appropriate laminar distribution of their branching dendritic trees. According to [11] the model exhibits behavioral regimes of normal brain activity that were not explicitly built-in but emerged spontaneously as the result of interactions among anatomical and dynamic processes.

To construct the Google matrix of brain we use a directed network of links between $N = 10^4$ neurons [30] generated from the brain model [11]. In total there are $N_l = 1960108$ links in the network. They form N_{out} *outgoing* links and N_{in} *ingoing* links

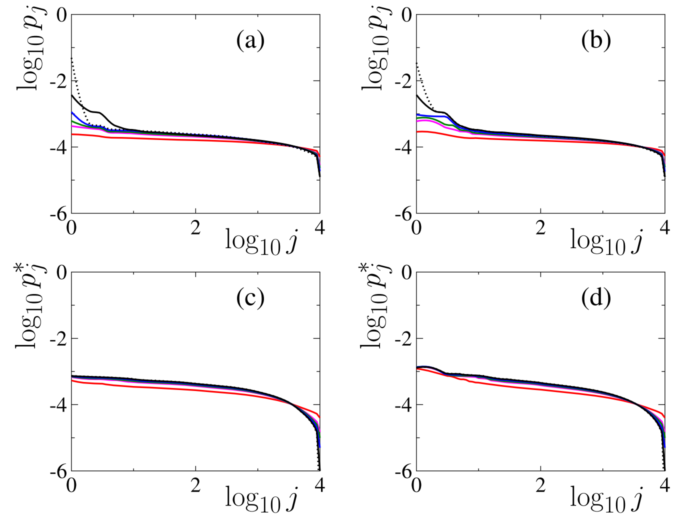

Fig. 2. (Color online.) PageRank p_j for the Google matrix of brain model at $\alpha = 0.6$, 0.85, 0.9, 0.95 and 0.99 shown by red, magenta green, blue and black solid curves (full curves from bottom to top at $\log_{10} j = 0.3$); j marks the index of nodes ordered according to the decreasing order of PageRank. The dotted black curve corresponds to $\alpha = 0.999$ and demonstrates strong dependence of the PageRank on α in the vicinity of $\alpha = 1$. Panels (a) and (b) correspond to unweighted and weighted links. For panels (a) and (b) the values of PAR are $\xi = 8223$ and 8314, 6295 and 6040, 5570 and 5046, 3283 and 3367, 28.4, 90.0, 1.09 and 1.19 for $\alpha = 0.6$, 0.85, 0.9, 0.95, 0.99, 0.999 respectively. Panels (c) and (d) show the dependence of the influence-PageRank $p^*(j)$ on j for the same values of α as for top panels respectively for unweighted and weighted links (for $\alpha > 0.6$ there is a strong overlap of curves).

($N_l = N_{out} = N_{in}$), so that there are about 200 outlinks (or ingoing) per neuron. Naturally, all links are counted as outgoing for one nodes and as ingoing for other nodes so that $N_{out} = N_{in} = N_l$. These numbers include multiple links between certain pairs of neurons; certain neurons have also links to themselves (there is one neuron linked only to itself). The number of weighted symmetric links is approximately 9.8%. Due to existence of multiple links between the same neurons we constructed two **G** matrices based on unweighted and weighted counting of links. In the first case all links from neuron j to neuron i are counted as one link, in the second case the weight of the link is proportional to the number of links from j to i. In both cases the sum of elements in one column is normalized to unity. The distributions of ingoing (P_{in}) and outgoing (P_{out}) links are shown in Fig. 1. The weighted distribution of ingoing links have a pronounced peaked structure corresponding to different regions of brain model considered in [11]. We note that the distribution of links is not of free-scale type.

The dependence of the PageRank on α is shown in Fig. 2. For $\alpha = 0.999$ almost all probability p_j is concentrated on one neuron. This is the only one neuron which is linked only to itself. With the increase of α up to 0.99 the main part of probability is concentrated mainly on about 10 neurons that approximately corresponds to the number of peaks in the distribution of weighted ingoing links in Fig. 1 (bottom left panel). At the same time the PageRank has a long tail at large j where the probability p_j is practically homogeneous. For $\alpha = 0.6$ the peak of probability at $1 \leqslant j \leqslant 10$ is washed out and the PageRank becomes completely delocalized. We note that a delocalization of the PageRank with α appears in the Ulam networks describing dynamical systems with dissipation [18,19]. At the same time the WWW networks remain stable in respect to variation of α as it is discussed in [25,31].

Recently, for the studies of procedure call network of the Linux Kernel [32] it was proposed to study the properties of the importance–PageRank $p^*(j)$ which is given by the eigenvector at