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Beyond the rotating wave approximation. An intensity dependent nonlinear coupling model in two-level systems

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ABSTRACT

An intensity dependent nonlinear coupling model of a two-level system interacting with a bimodal cavity field via two-photon transitions is investigated in a scenario where the rotating wave approximation is lifted. The model is numerically tested against simulations of normal squeezing variance and entropy squeezing factors based on the Heisenberg uncertainty principle and Shannon information theory derived from entangled states.

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1. Introduction

The matter–radiation interaction is a central problem in quantum optics. The simplest model to deal with this is the Rabi model [1], which describes the interaction of a two-level atom with a single mode of the quantized electromagnetic field. Although widely studied over the past few decades, up to now an exact analytical solution is lacking and only numerical [2,3] and approximate analytical solutions are available [4,5], despite the conjecture by Reik and Doucha [6,7] that an exact solution of the Rabi Hamiltonian in terms of known functions is possible. The commonest analytical approach to solving the Rabi model is to make use of the rotating wave approximation (RWA), where the counter-rotating terms are neglected. In this limit, the Rabi Hamiltonian is known as Jaynes–Cummings (JC) Hamiltonian and can be integrated exactly [8,9]. In spite of the simplicity of the JC model, the dynamics have turned out to be very rich and complex. In fact, this model has revealed interesting phenomena related to the quantum nature of the light, encompassing the granular nature of the electromagnetic field, revealed through the existence of nonclassical effects such as revival of the atomic inversion, Rabi oscillations, squeezing [10,11], and atom–atom or atom–field entanglement [12].

The manipulation of atom–field interaction has been employed in cavity quantum electrodynamics, as well as in the atomic teleportation process, which have contributed to a fast development of quantum information theory [13]. With the experimental progress of some systems it was found that the coupling between the systems may be made very large, and the RWA breaks down so that only the Rabi model describes the dynamics correctly [14]. In addition, recent papers have questioned the validity of the RWA [14,15] and proposed alternative analytical approximate methods [4,5]. Moreover, it has been shown that the counter-rotating terms are responsible for several novel quantum-mechanical phenomena [16].

Strict analysis of the validity of the RWA is not usually considered in concrete applications, and the range of system parameters where the results are meaningful remains uncertain. Therefore, it is of great interest to explore approximate solutions of more complex Hamiltonians that contain the counter-rotating terms for a wide range of the system parameters, and compare them with the RWA results. Such studies are useful for determining the limits of validity of the JC models. In the present work the counter-rotating terms are explicitly taken into consideration along with a simultaneous inclusion of an intensity dependent coupling in the bosonic part of the interacting Hamiltonian. Further incorporation of two

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laser fields with different amplitudes and frequencies in the quantum regime involving two-photon transitions generates in essence a highly nonlinear Hamiltonian. The previously uninvestigated resulting two-level model will be tackled through time-dependent perturbation expansion of the evolution operator. Thus, density operator matrix elements which are necessary to compute different nonclassical properties arising from an excited state and its decay to the ground state in the two-level system are to be computed through first- and second-order of the chronologically ordered time-dependent perturbation expansion of the interaction picture evolution operator for the ground and excited states, respectively. Such a generalization is of considerable interest because of its relevance to the study of the coupling between a single atom and the radiation field with the atom making k-photon transitions. To illustrate the applicability of the model numerical simulations of the normal squeezing variance and entropy squeezing factors based on the Heisenberg uncertainty principle and Shannon information theory derived from entangled states are presented.

2. Atom–cavity interaction generalized model

Let us consider a bosonic system S, with Hilbert space $\mathcal{S}^{(S)}$ which is coupled with a two-level atom, with Hilbert space $\mathcal{S}^{(B)}$. Let us assume that the complete system is in thermal equilibrium with a reservoir at temperature β^{-1}. It is important to keep in mind that the presence of the reservoir only takes the atom (or in fact, any other two-level system) and the bosonic modes in thermal equilibrium. Let us denote by $\mathcal{H}_S$, $\mathcal{H}_B$, and $\mathcal{H}_I$ the Hamiltonians of the bosonic field, the two-level atom, and the interaction between both systems, respectively. The Hamiltonian for the total system can be written as

$$\mathcal{H} = \mathcal{H}_S \otimes I_B + I_S \otimes \mathcal{H}_B + \mathcal{H}_I \equiv \mathcal{H}_0 + \mathcal{H}_I, \tag{2.1}$$

where I_S and I_B denote the identities in the Hilbert spaces of the bosonic field and the atomic system.

The aim of this section is to introduce the necessary formalism to develop generalized models of two-level systems in which the counter-rotating terms are not ignored and an intensity dependent nonlinear coupling is explicitly incorporated in the Hamiltonian. It is further assumed that the electromagnetic field is associated with two modes of monocromatic radiation and induces two-photon transitions. This makes the present model to be strongly nonlinear and therefore only approximate solutions can be developed. The system dynamics will be explored through the density operator formalism emerging from the chronologically ordered time-dependent perturbation Dyson expansion. The complete Hamiltonian for such model reads

$$\begin{aligned}\mathcal{H} &= \hbar \sum_{j=1}^{2} \nu_j a_j^\dagger a_j \otimes I_B + I_S \otimes \hbar/2 \omega \sigma_z \\ &\quad + \hbar \sum_{j=1}^{2} g_j \big(R_j^k + R_j^{\dagger k}\big) \otimes \sigma_x \equiv \mathcal{H}_0 + \mathcal{H}_I,\end{aligned} \tag{2.2}$$

where ν_j and g_j are the photon frequency and atom–field coupling constant (vacuum Rabi frequency) for the mode j respectively; $a_j^\dagger$ (a_j) is the creation (annihilation) bosonic operator for the mode j, the zero-point energy of the bosonic field was omitted, and a constant term $1/2(\omega_a + \omega_b)$, where ω_a and ω_b are the energies of the ground ($|a\rangle$) and excited ($|b\rangle$) states of the two-level system, was ignored. As usual, we are using the pseudo-spin operators $\sigma^+ = |b\rangle\langle a|$, $\sigma^- = |a\rangle\langle b|$, $\sigma_x = |b\rangle\langle a| + |a\rangle\langle b|$, and $\sigma_z = |b\rangle\langle b| - |a\rangle\langle a|$ for the two-level atom which satisfy the standard angular momentum commutation relations corresponding to spin 1/2 Pauli operators, and therefore, they constitute a basis of the $SU(2)$ algebra. Thus, these atom-flip operators characterize the effective two-level system with transition frequency $\omega = \omega_b - \omega_a$. In Eq. (2.2) R_j^k and $R_j^{\dagger k}$ are intensity dependent *shifting* operators involving k photons, i.e.,

$$R_j^k = a_j^k \big(a_j^\dagger a_j\big)^{1/2}, \tag{2.3}$$

and its Hermitian conjugate

$$R_j^{\dagger k} = \big(a_j^\dagger a_j\big)^{1/2} a_j^{\dagger k}. \tag{2.4}$$

In the present generalized model the counter-rotating terms $R_j^k \sigma^- + R_j^{\dagger k} \sigma^+$, ignored under WRA, are retained in the Hamiltonian (2.2). It is convenient to work in the interaction picture with the Hamiltonian given by

$$\mathcal{V}(t) = \mathcal{U}_0^\dagger(t) \mathcal{H}_I \mathcal{U}_0(t), \tag{2.5}$$

where $\mathcal{U}_0$ is the unitary time evolution operator for the unperturbed Hamiltonian and which merely contributes a phase factor in each atomic subspace. Using the expansion

$$\exp(\alpha A) B \exp(-\alpha A) = B + \alpha[A, B] + \alpha^2/2! \big[A, [A, B]\big] + \cdots, \tag{2.6}$$

along with the commutation relations

$$\big[a_j^\dagger a_j, R_i^k\big] = -k R_i^k \delta_{ij}, \tag{2.7}$$

$$\big[a_j^\dagger a_j, R_i^{\dagger k}\big] = k R_i^{\dagger k} \delta_{ij}, \tag{2.8}$$

and consequently noting that

$$\exp\big(i\nu_j a_j^\dagger a_j t\big) R_j^k \exp\big(-i\nu_j a_j^\dagger a_j t\big) = R_j^k \exp(-i\nu_j k t), \tag{2.9}$$

$$\exp\big(i\nu_j a_j^\dagger a_j t\big) R_j^{\dagger k} \exp\big(-i\nu_j a_j^\dagger a_j t\big) = R_j^{\dagger k} \exp(i\nu_j k t), \tag{2.10}$$

$$\exp(i\omega t \sigma_z/2) \sigma_x \exp(-i\omega t \sigma_z/2) = \begin{pmatrix} 0 & \exp(i\omega t) \\ \exp(-i\omega t) & 0 \end{pmatrix}, \tag{2.11}$$

the interaction picture Hamiltonian (2.5) can be written as

$$\mathcal{V}(t) = \hbar \begin{pmatrix} 0 & \exp(i\omega t) \\ \exp(-i\omega t) & 0 \end{pmatrix} \sum_{j=1}^{2} g_j \big(R_j^k \exp(i\Delta_j t) + \text{h.c.}\big), \tag{2.12}$$

with the detuning parameter Δ_j for the mode j given by

$$\Delta_j = \omega - k\nu_j. \tag{2.13}$$

These detunings between the cavity mode and the atomic transition can have an important influence on the nonclassical effects, as recently reported in the case of a two-level atom coupled to a single mode of cavity fields [17].

The dynamics of the present model is not stationary and depends on the initial conditions of the system and the cavity field. Thus, it is assumed that, initially, the field modes are in coherent states and the atomic system is in the excited state $|b\rangle$, that is, the atomic system and the field are initially in a disentangled state. It is further assumed that at $t = 0$, the two modes have the same photon distribution with density operator

$$\begin{aligned}\rho(0) &= \big|\psi(0)\big\rangle\big\langle\psi(0)\big| \\ &= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} c_{n_1 n_2}(0) c_{m_1 m_2}^*(0) |b; m_1 m_2\rangle\langle b; n_1 n_2|\end{aligned} \tag{2.14}$$

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