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A general method for modified function projective lag synchronization in chaotic systems ☆

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ABSTRACT

The modified function projective lag synchronization (MFPLS) is proposed in this Letter, in which the states of two chaotic systems are asymptotically lag synchronized up to a desired scaling function matrix. Based on Lyapunov stability theory, a general method of MFPLS is investigated. The scheme is successfully applied to two groups examples, which are the MFPLS between Lorenz system and Lü system, and two identical hyper-chaotic Chen system. Corresponding numerical simulations are performed to verify and illustrate the analytical results.

1. Introduction

Chaos synchronization is a hot subject in the field of nonlinear science due to its wide-scope potential applications in physical systems, biological networks, secure communications, etc. Since the pioneering works by Pecora and Carroll [1], in which proposed a successful method to synchronize two identical chaotic systems with different initial conditions, various types of synchronization phenomena have been revealed to synchronize chaotic systems [2–6]. Amongst all kinds of chaos synchronization, projective synchronization [6] has received much attention [7–11] as it can obtain faster communication with its proportional feature.

Considered a time delay will affect the projective synchronization of chaotic systems, some authors [12–14] study the problem of projective synchronization with time delay. Projective lag synchronization was proposed by Li in [12], where a driven chaotic system synchronizes the past state of the driver up to a scaling factor α. Namely, the response system's output lags behind the output of the driver system proportionally. Ref. [13] investigated the problem of full state hybrid lag projective synchronization in chaotic systems. Ref. [14] reported on generalized projective synchronization between two identical time delay chaotic systems with single time delays.

In recent years, another regime of projective synchronization called function projective synchronization (FPS) was also extensively investigated [15–19], in which the drive and response systems can synchronize up to a desired scaling function. Because the unpredictability of the scaling function in FPS can additionally enhance the security of communication. Recently a more general form of function projective synchronization, which is named as modified function projective synchronization (MFPS), has been proposed in [20]. MFPS means the master and slave systems could be synchronized up to a scaling function matrix. The novelty feature of this synchronization phenomenon is that the scaling functions can be arbitrarily designed to different state variables by means of control.

Motivated by the existing works and take into account of the time delay, we investigate a new type of synchronization phenomenon, modified function projective lag synchronization (MFPLS), which makes the states of two chaotic systems asymptotically lag synchronized up to a desired scaling function matrix. Furthermore, a general method is presented to realize MFPLS. To the best of our knowledge, at present, there are few theoretical results about MFPLS.

The organization of this Letter is as follows: In Section 2, the definition of MFPLS is given. In Section 3, a general scheme of

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MFPLS is presented in two chaotic systems. In Section 4, two groups of examples are used to verify the effectiveness of the proposed scheme. The conclusion is finally drawn in Section 5.

2. The definition of MFPLS

The drive system and the response system are defined below

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}\big(\boldsymbol{x}(t)\big), \tag{1}$$

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{g}\big(\boldsymbol{y}(t)\big) + \boldsymbol{u}(t), \tag{2}$$

where $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{R}^n$ are the state vectors, $\boldsymbol{f}, \boldsymbol{g} : \boldsymbol{R}^n \to \boldsymbol{R}^n$ are continuous nonlinear vector functions, $\boldsymbol{u}(t)$ is the vector controller. We define the error vector

$$\boldsymbol{e}(t) = \boldsymbol{x}(t-\tau) - \boldsymbol{\Lambda}(t)\boldsymbol{y}(t), \tag{3}$$

where $\boldsymbol{\Lambda}(t) = \mathrm{diag}(\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t))$ is reversible and differentiable, where $\alpha_i(t) \neq 0$ is a continuously differentiable function with bounded.

Definition 1 *(MFPLS).* For the drive system (1) and the response system (2), it is said that the system (1) and the system (2) are modified function projective lag synchronization, if there exist a delay time τ and a scaling function matrix $\boldsymbol{\Lambda}(t)$ such that $\lim_{t\to\infty} \|\boldsymbol{x}(t-\tau) - \boldsymbol{\Lambda}(\boldsymbol{t})\boldsymbol{y}(t)\| = \boldsymbol{0}$.

3. A general method for MFPLS

The drive and response systems are defined as Eqs. (1) and (2). Our goal is to make MFPLS between the drive system (1) and the response system (2) by designing the control law $\boldsymbol{u}$, i.e. $\lim_{t\to\infty} \|\boldsymbol{e}(t)\| = 0$.

Theorem 1. *For given a synchronization scaling function matrix $\boldsymbol{\Lambda}(t)$, a delay time τ and any initial conditions $\boldsymbol{x}(0)$, $\boldsymbol{y}(0)$, the MFPLS between the drive system (1) and the response system (2) will occur by the control law (4) as below*

$$\boldsymbol{u}(t) = \boldsymbol{\Lambda}^{-1}(t)\big[\boldsymbol{f}\big(\boldsymbol{x}(t-\tau)\big) - \boldsymbol{\Lambda}(t)\boldsymbol{g}\big(\boldsymbol{y}(t)\big) - \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{y}(t) + \boldsymbol{e}(t)\big], \tag{4}$$

where $\boldsymbol{e}(t) = \boldsymbol{x}(t-\tau) - \boldsymbol{\Lambda}(t)\boldsymbol{y}(t)$.

Proof. The time derivative of the error vector (3) is

$$\dot{\boldsymbol{e}}(t) = \dot{\boldsymbol{x}}(t-\tau) - \boldsymbol{\Lambda}(t)\dot{\boldsymbol{y}}(t) - \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{y}(t). \tag{5}$$

Substituting (1) and (2) into (5), we have

$$\dot{\boldsymbol{e}}(t) = \boldsymbol{f}\big(\boldsymbol{x}(t-\tau)\big) - \boldsymbol{\Lambda}(t)\boldsymbol{g}\big(\boldsymbol{y}(t)\big) - \boldsymbol{\Lambda}(t)\boldsymbol{u}(t) - \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{y}(t). \tag{6}$$

Construct Lyapunov function

$$\boldsymbol{V} = \frac{1}{2}\boldsymbol{e}^T(t)\boldsymbol{e}(t). \tag{7}$$

With the choice of the controller (4), the time derivative of V along the trajectories of Eq. (6) is

$$\begin{aligned}
\dot{\boldsymbol{V}} &= \boldsymbol{e}^T(t)\dot{\boldsymbol{e}}(t) \\
&= \boldsymbol{e}^T(t)\big[\boldsymbol{f}\big(\boldsymbol{x}(t-\tau)\big) - \boldsymbol{\Lambda}(t)\boldsymbol{g}\big(\boldsymbol{y}(t)\big) - \boldsymbol{\Lambda}(t)\boldsymbol{u}(t) - \dot{\boldsymbol{\Lambda}}(t)\boldsymbol{y}(t)\big] \\
&= -\boldsymbol{e}^T(t)\boldsymbol{e}(t).
\end{aligned} \tag{8}$$

It is clear that $\boldsymbol{V}$ is positive definite and $\dot{\boldsymbol{V}}$ is negative definite. According to the Lyapunov stability theory, the error vector $\boldsymbol{e}(t)$ asymptotically tends to zero leading to MFPLS occur. This completes the proof. □

Remark 1. Note that Theorem 1 can also encompass the MFPLS scheme of two identical chaotic systems when the vector function $\boldsymbol{g} = \boldsymbol{f}$ in Eq. (4).

Remark 2. Note that Theorem 1 just given a general scheme of MFPLS in theory, according to the characters of the practical synchronized systems, the complexity of the controller could be reduced, appropriately. How to derive a simple general controller for synchronizing any two chaotic systems is needed to further investigate.

4. Illustrative example

In this section, we will give two groups of examples to verify the effectiveness of the proposed scheme, which are chaotic Lorenz and Lü systems, and two identical hyper-chaotic Chen systems. Numerical simulations are performed to demonstrate the effectiveness of the proposed method.

4.1. MFPLS between chaotic Lorenz and Lü systems

We take chaotic Lorenz system as the drive system, which is described by the following formulas

$$\begin{cases}
\dot{x}_d = 10(y_d - x_d), \\
\dot{y}_d = (28 - z_d)x_d - y_d, \\
\dot{z}_d = x_d y_d - 8/3 z_d,
\end{cases} \tag{9}$$

where x_d, y_d and z_d are state variables.

The chaotic Lü system, as a response system, is given by

$$\begin{cases}
\dot{x}_r = 36(y_r - x_r) + u_1, \\
\dot{y}_r = -x_r z_r + 20 y_r + u_2, \\
\dot{z}_r = x_r y_r - 3 z_r + u_3,
\end{cases} \tag{10}$$

where x_r, y_r and z_r are state variables, u_1, u_2 and u_3 are the nonlinear control laws such that two chaotic systems can be synchronized in the sense of MFPLS.

We define the MFPLS errors as

$$\begin{cases}
e_1(t) = x_d(t-\tau) - \alpha_1(t) x_r(t), \\
e_2(t) = y_d(t-\tau) - \alpha_2(t) y_r(t), \\
e_3(t) = z_d(t-\tau) - \alpha_3(t) z_r(t),
\end{cases} \tag{11}$$

where $\alpha_1(t)$, $\alpha_2(t)$ and $\alpha_3(t)$ are the scaling functions.

According to Eq. (4) in Theorem 1, we get the controller

$$\boldsymbol{u} = \begin{bmatrix}
\frac{10(y_{d\tau} - x_{d\tau}) - 36\alpha_1(t)(y_r - x_r) - \dot{\alpha}_1(t) x_r + e_1}{\alpha_1(t)} \\
\frac{(28 - z_{d\tau}) x_{d\tau} - y_{d\tau} - \alpha_2(t)(-x_r z_r + 20 y_r) - \dot{\alpha}_2(t) y_r + e_2}{\alpha_2(t)} \\
\frac{x_{d\tau} y_{d\tau} - 8 z_{d\tau}/3 - \alpha_3(t)(x_r y_r - 3 z_r) - \dot{\alpha}_3(t) z_r + e_3}{\alpha_3(t)}
\end{bmatrix}, \tag{12}$$

where $x_{d\tau} = x_d(t-\tau)$, $y_{d\tau} = y_d(t-\tau)$, $z_{d\tau} = z_d(t-\tau)$, $x_r = x_r(t)$, $y_r = y_r(t)$, $z_r = z_r(t)$.

To verify and show the effectiveness of the controller (12), fourth-order Runge–Kutta method is used with time step being equal to 0.001. In numerical simulations, we assume that $\alpha_1(t) = \sin(t) + 2$, $\alpha_2(t) = \sin(t) + 3$, $\alpha_3(t) = \sin(t) + 4$ and $\tau = 2$. The simulation results are shown in Figs. 1 and 2.

The time evolution of the MFPLS errors are depicted in Fig. 1, which displays $\boldsymbol{e} \to 0$ with $t \to \infty$. Thus, the required synchronization has been achieved with our designed control law (12). The scaling functions $\alpha_1(t)$, $\alpha_2(t)$ and $\alpha_3(t)$ are depicted in Fig. 2, which tend to the predefined scaling functions. These results show that MFPLS takes place with the desired scaling function matrix.