



Modulational instability in two cubic–quintic Ginzburg–Landau equations coupled with a cross phase modulation term

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ABSTRACT

Modulation instability is investigated in two cubic–quintic Ginzburg–Landau equations coupled with a cross phase modulation type term. After carrying out a stability analysis an expression for gain is obtained. Some direct simulations to see the evolution of different continuous wave states are reported. These show the formation of modulation instability pulses as well as transitions from lower amplitude continuous wave states to higher amplitude continuous wave states.

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1. Introduction

A continuous wave (CW) is subject to modulational instability (MI) under the action of non-linearity in combination with dispersion. As result of MI a continuous wave propagating in a nonlinear dispersive medium may suffer the instability with respect to weak periodic modulations of the steady-state. Due to the interaction between a strong harmonic carrier wave and small sidebands displaced a little in frequency get to the breakup of CW into a train of ultra short pulses [1,2]. The development of the theory of MI started in the 1960s, simultaneously to hydrodynamics and nonlinear optics. MI was also observed in different physical systems such as fluid dynamics or plasma physics [3,4]. A recent review about the beginning of MI can be found in [2]. In nonlinear optics MI has been studied for beam propagation in optical fibers using scalar and vectorial descriptions. In a scalar description MI occurs only in anomalous dispersion regime while in a vectorial description occurs also in the normal dispersion regime [5,6].

Our principal aim is to investigate the properties of the MI of two complex Ginzburg–Landau equations (CGLE) with both cubic and quintic terms coupled with a cubic term. The CGLE is a generic equation describing systems near subcritical bifurcations related to a wide range of dissipative phenomena in physics [7]. CGLE have been used to simulate pulse propagation in fibers including higher order effects [8,9].

The CGLE is an extension of the nonlinear Schrödinger equation (NLSE) including terms for nonlinear gain and saturation of the nonlinear refractive index. The propagation of vectorial pulses

in birefringence fibers could be modelled using two NLSE coupled with a cross-phase modulation (XPM) term [10,5]. An extension of this model uses two cubic–quintic nonlinear Schrödinger equations (CQNLS) with a coupling term obtained from an interaction Hamiltonian that takes into account both the XPM and higher-order terms, which only produce a small correction to the XPM term [11]. According to this, a system of two CGLE coupled with a XPM term is an extension of those models, to include spectral filtering or parabolic gain, nonlinear gain or absorption processes and higher-order correction to the nonlinear amplification/absorption and the intensity-dependent refractive index. Thus, we will consider a symmetric system of equations for the amplitudes U and V of the two components of a field in a birefringence fiber:

$$\frac{\partial U}{\partial z} = \left(\beta + i \frac{D}{2} \right) \frac{\partial^2 U}{\partial t^2} - \alpha \frac{\partial U}{\partial t} + \delta U + (\epsilon + i)|U|^2 U + (\mu + i\nu)|U|^4 U + iB|V|^2 U \quad (1)$$

$$\frac{\partial V}{\partial z} = \left(\beta + i \frac{D}{2} \right) \frac{\partial^2 V}{\partial t^2} + \alpha \frac{\partial V}{\partial t} + \delta V + (\epsilon + i)|V|^2 V + (\mu + i\nu)|V|^4 V + iB|U|^2 V \quad (2)$$

where D , δ , β , ϵ , μ , α , and ν are real constants, U and V are complex fields. By a proper rescaling and without lossing generality D can be restricted to have the values $D = \pm 1$. When used to describing propagation of light in optical fiber, t is a retarded time, z is the propagation distance, ψ is the complex envelope of the electric field, D is the dispersion coefficient, δ gives account of the linear gain, β describes spectral filtering or parabolic gain (if $\beta > 0$), ϵ refers to nonlinear gain or absorption processes,

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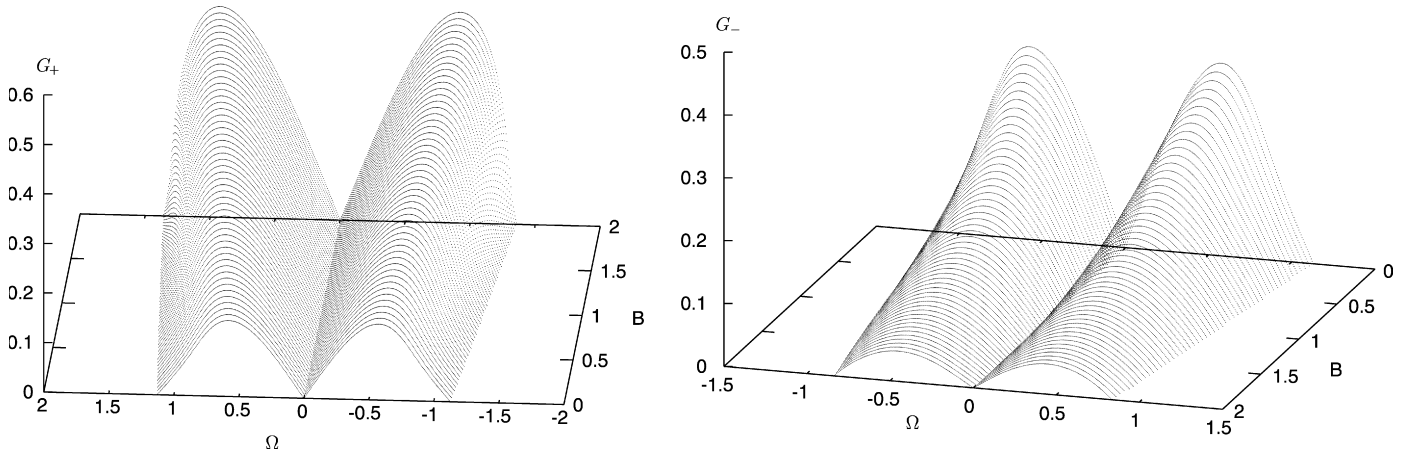


Fig. 1. G_+ and G_- as functions of Ω and B for the solution set (A_l, C_l, ω_l) and the system parameters: $\beta = 1$, $\epsilon = 0.7$, $\mu = -0.01$, $\nu = -0.1$ and $\delta = -0.1$.

μ represents a higher-order correction to the nonlinear amplification/absorption, and ν is a higher order correction term to the intensity-dependent refractive index [8]. B is the coupling parameter and α governs the group-velocity mismatch between the two components. In order to a simplest description we are going to consider only the case $\alpha = 0$. In this situation, $B = 2$ corresponds to low-birefringence fibers. Stable soliton solutions of the CGLE in one dimension exists only for the following choices of the signs of the coefficients: $\delta, \mu < 0$, $\beta, \epsilon > 0$, and any sign for ν and D [12]. Since MI can develop stable pulses we will restrict to the above choices of the signs of the coefficients.

The Letter is organised as follows: in Section 2, we obtain the analytic expressions for the gain by the MI. In Section 3, we simulate the dynamics of the initial steady CW states under the weak modulational instability. The conclusions are followed in Section 4.

2. Stability analysis and MI gain

In order to investigate the evolution of weak perturbations along the propagation distance, we carry out a linear-stability analysis. First, we look for a CW solution of the system described by (1) and (2) given by:

$$U = Ae^{i\omega z} \quad (3)$$

$$V = Ce^{i\omega z} \quad (4)$$

By substituting in Eqs. (1) and (2) with t -derivative terms equal to zero, requiring both imaginary and real terms to be zero, we obtain:

$$\begin{aligned} \delta + (\epsilon + i)A^2 + (\mu + i\nu)A^4 - i\omega + iBC^2 &= 0 \\ \delta + (\epsilon + i)C^2 + (\mu + i\nu)C^4 - i\omega + iBA^2 &= 0 \end{aligned} \quad (5)$$

which yields four sets of stationary solutions. Two of them matches with what we call symmetric case, where $A = C$:

$$\begin{aligned} A_{l,h}^2 = C_{l,h}^2 &= \frac{-\epsilon \pm \sqrt{\epsilon^2 - 4\mu\delta}}{2\mu} \\ \omega &= C_{l,h}^2 \left(1 + B - \epsilon \frac{\nu}{\mu} \right) - \frac{\delta\nu}{\mu} \end{aligned} \quad (6)$$

where the subscripts l (for low) and h (for high) correspond to the + and - signs, respectively. Thus, $A_l < A_h$.

The other two sets correspond to the asymmetric case, where $A \neq C$. These solutions can be written as (A_l, C_h, ω) and (A_h, C_l, ω) where A_l, A_h, C_l, C_h and ω are given by Eqs. (6).

We note that the solution sets are functions of the system parameters except for the spectral filtering term β . This also occurs in systems described by two CGLE coupled with a linear term [13]. Moreover, A (or C) values are the same as those for 1D CGLE. As reported by [14], A_l (or C_l) is unstable whereas A_h (or C_h) is stable. As the XPM term has no contribution to the gain, we think that for symmetric case, the solution (A_l, C_l, ω) becomes unstable whereas (A_h, C_h, ω) remains stable.

In order to examine the linear-stability of the CW states we introduce the perturbed fields of the form

$$\begin{aligned} U(z, t) &= [A + u(z, t)]e^{i\omega z} \\ V(z, t) &= [C + v(z, t)]e^{i\omega z} \end{aligned} \quad (7)$$

where $|u(z, t)| \ll A, C$ and $|v(z, t)| \ll A, C$. Substituting in Eqs. (1) and (2) and retaining linear terms of u and v , we obtain two linearized equations:

$$\begin{aligned} \frac{\partial u}{\partial z} &= \left(\beta + i \frac{D}{2} \right) \frac{\partial^2 u}{\partial t^2} \\ &+ (\delta + 2\epsilon A^2 + 3\mu A^4 + i(2A^2 - \omega + 3\nu A^4 + BC^2))u \\ &+ (\epsilon A^2 + \mu 2A^4 + i(A^2 + 2\nu A^4))u^* + iBC^*Av + iBCAv^* \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{\partial v}{\partial z} &= \left(\beta + i \frac{D}{2} \right) \frac{\partial^2 v}{\partial t^2} \\ &+ (\delta + 2\epsilon C^2 + 3\mu C^4 + i(2C^2 - \omega + 3\nu C^4 + BA^2))v \\ &+ (\epsilon C^2 + \mu 2C^4 + i(C^2 + 2\nu C^4))v^* + iBA^*Cu + iBACu^* \end{aligned} \quad (9)$$

where $*$ denotes complex conjugate. We assume general solutions of the form

$$\begin{aligned} u(z, t) &= a_1 e^{i(kz - \Omega t)} + a_2 e^{-i(kz - \Omega t)} \\ v(z, t) &= b_1 e^{i(kz - \Omega t)} + b_2 e^{-i(kz - \Omega t)} \end{aligned} \quad (10)$$

where k and Ω represent the wave number and the frequency of modulation, respectively. Inserting Eq. (10) to Eqs. (8) and (9), we obtain four equations to a_1, a_2, a_3 and a_4 . For the symmetric case, these equations are:

$$\begin{aligned} ika_1 &= \left(\beta + i \frac{D}{2} \right) (-\Omega^2) a_1 + (\lambda_1 + i\lambda_2)(a_1 + a_2) \\ &+ iBA^2(b_1 + b_2) \end{aligned}$$

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