Physics Letters A 373 (2009) 2682–2694

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Global synchronization in arrays of coupled networks with one single time-varying delay coupling ☆

Wangli He *, Jinde Cao

Department of Mathematics, Southeast University, Nanjing 210096, China

ARTICLE INFO

Article history:
Received 26 November 2008
Received in revised form 5 May 2009
Accepted 18 May 2009
Available online 22 May 2009
Communicated by A.R. Bishop

PACS:
05.45.Xt
05.45.Vx
05.45.Gg

Keywords:
Global synchronization
Complex networks
Lyapunov functional
One single delay

ABSTRACT

Global synchronization in arrays of coupled networks with one single time-varying delay coupling is investigated in this Letter. A general linear coupled network with a time-varying coupling delay is proposed and its global synchronization is further discussed. Some sufficient criteria are derived based on Lyapunov functional and linear matrix inequality (LMI). It is shown that under one single delay coupling, the synchronized state changes, which is different from the conventional synchronized solution. Moreover, the degree of the nodes and the inner delayed coupling matrix play key roles in the synchronized state. In particular, the derivative of the time-varying delay can be any given value. Finally, numerical simulations are given to illustrate the theoretical results.

1. Introduction and model description

Networks are ubiquitous in the real world, such as food-webs, ecosystems, metabolic pathways, Internet, World Wide Web, social networks, and global economic markets. Recent years have witnessed an increasing interest in complex networks from the science and technology communities [1–3]. Among them, synchronization is the most interesting and has been extensively investigated [4–26].

Recently, synchronization in arrays of coupled dynamical networks has attracted increasing attention in various research fields [7–26]. In [10,11], the authors proposed a master stability function based on the transverse Lyapunov exponents to study local synchronization. In [7–9,14–20], the authors obtained some sufficient conditions ensuing local synchronization via linearization technique. Global synchronization of coupled systems has also been investigated in [21–26] by different ways. In [8,9], the authors considered a dynamical network with linear diffusive coupling, which was described by

$$\dot{x}_i(t) = f\big(x_i(t)\big) + c \sum_{j=1,\, j\neq i}^{N} G_{ij}\Gamma\big(x_j(t) - x_i(t)\big), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ denotes the state variable vector of the ith node. $f(.) : \mathbb{R}^n \to \mathbb{R}^n$ is a differentiable function. The scale c is the coupling strength. $G = (G_{ij})_{N\times N}$ is the coupling configuration matrix representing the topological structure of the network. $G_{ij} \geqslant 0$ and satisfies $G_{ii} = -\sum_{j=1,\, j\neq i}^{N} G_{ij} = -\sum_{j=1,\, j\neq i}^{N} G_{ji}$. The n-dimensional diagonal matrix $\Gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ is inner coupling matrix. Later, Lü and Chen [14] introduced a time-varying model with the same prototype of (1) and discussed its local synchronization. Some recent works [15,16] also were devoted to the time-varying coupling.

☆ This work was jointly supported by the National Natural Science Foundation of China under Grant No. 60874088, the Specialized Research Fund for the Doctoral Program of Higher Education under Grant 20070286003, and the Foundation for Excellent Doctoral Dissertation of Southeast University YBJJ0806.

* Corresponding author. Fax: +86 25 83792316.
E-mail addresses: wanglihe07@gmail.com (W. He), jdcao@seu.edu.cn (J. Cao).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2009.05.041

Time delay is inevitable in the signal transmission process due to finite speeds of transmission and spreading as well as traffic congestions. Therefore, Li, Zhou and Chen [17–19] extended the model with delayed coupling:

$$\dot{x}_i(t) = f\big(x_i(t)\big) + c\sum_{j=1,\, j\neq i}^{N} G_{ij}\Gamma\big(x_j(t-\tau) - x_i(t-\tau)\big), \quad i = 1, 2, \ldots, N. \tag{2}$$

Synchronization of system (2) with time-varying delay was addressed in [20].

Considering the wide application of neural networks, a linear coupled neural network consisting of N is proposed as follows:

$$\dot{x}_i = -Cx_i(t) + Af\big(x_i(t)\big) + Bf\big(x_i(t-\tau)\big) + J(t) + \sum_{j=1,\, j\neq i}^{N} G_{ij}\Gamma\big(x_j(t) - x_i(t)\big). \tag{3}$$

In [21–23], the authors studied global synchronization of system (3) by introducing a distance to synchronization with some structural matrix. Later, Cao and his colleagues [24,25] further extended the model as:

$$\begin{aligned}\dot{x}_i &= -Cx_i(t) + Af\big(x_i(t)\big) + Bf\big(x_i\big(t-\tau(t)\big)\big) + J(t) + \sum_{j=1,\, j\neq i}^{N} G_{ij}D\big(x_j(t) - x_i(t)\big)\\ &\quad + \sum_{j=1,\, j\neq i}^{N} G_{ij}D_\tau\big(x_j\big(t-\tau(t)\big) - x_i\big(t-\tau(t)\big)\big).\end{aligned} \tag{4}$$

Different methods are applied to investigate global synchronization of coupled networks (4) when $\tau(t)$ is constant or time-varying, respectively. Distributed delay coupling are also studied in [26].

It is well known that communication delays exist extensively in networks. Therefore, it is reasonable to assume that it exists delay in the information transmission. If there is a connection from node j to node i. The information, received by node i is with time delay. That is, the delay affects only the variable that is being transmitted from one system to another. Therefore, it makes sense to assume that there is only one single delay in the coupling part. This communication scheme can be found in the consensus problem [27], also in synchronization of coupled chaotic maps [28]. Lu and Chen [29] proposed a general coupled systems with a coupling delay, which was given by

$$\dot{x}_i(t) = f\big(x_i(t)\big) + c\sum_{j=1,\, j\neq i}^{N} a_{ij}\Gamma\big(x_j(t-\tau) - x_i(t)\big), \quad i = 1, 2, \ldots, N. \tag{5}$$

They both discussed its local and global synchronization. A similar linear model was reported in [30] and some recent work [31] has been done in the distributed-delay case.

In fact, time delay is not necessary to be constant in many real-world networks. And neural networks have wild applications. Therefore, a new coupled neural networks is proposed, which is described by

$$\begin{aligned}\dot{x}_i(t) &= -Cx_i(t) + Af\big(x_i(t)\big) + Bf\big(x_i\big(t-\tau(t)\big)\big) + J(t) + \sum_{j=1,\, j\neq i}^{N} G_{ij}D\big(x_j(t) - x_i(t)\big)\\ &\quad + \sum_{j=1,\, j\neq i}^{N} G_{ij}D_\tau\big(x_j\big(t-\tau(t)\big) - x_i(t)\big), \quad i = 1, 2, \ldots, N,\end{aligned} \tag{6}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ denotes the state variable associated with the ith node; $C = \operatorname{diag}\{c_1, c_2, \ldots, c_n\}$ is the decay constant matrix with $c_i > 0$, $i = 1, 2, \ldots, n$, $f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)), \ldots, f_n(x_{in}(t)))$ is the activation function of the neurons, $\tau(t)$ is the transmission delay and satisfy $0 \leqslant \tau(t) \leqslant \tau$ (τ is a positive constant). $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are the connection matrix and delayed connection matrix, respectively, $J = (J_1(t), J_2(t), \ldots, J_n(t)) \in \mathbb{R}^n$ is an external input vector. $D = (d_{rj})_{n\times n}$ and $D_\tau = (d^\tau_{rj})_{n\times n}$ are respectively the inner coupling matrices between the connected nodes i and j at time t and $t - \tau(t)$; $G = (G_{ij})_{N\times N}$ is the configuration matrix that is irreducible and satisfies the following conditions:

$$G_{ij} = G_{ji} \geqslant 0, \quad i \neq j, \qquad G_{ii} = -\sum_{j=1,\, j\neq i}^{N} G_{ij}. \tag{7}$$

$G_{ij} > 0$ if there is a connection between node i and node j and $G_{ij} = 0$ otherwise. The degree of node i is equal to $\sum_{j=1,\, j\neq i}^{N} G_{ij}$.

In particular, the derivative of the time-varying delay can be any given value. Some papers [21–24] discussed global synchronization with constant delay or time-varying delay, whose derivative should be less than 1. In this Letter, we discuss global synchronization of linearly coupled neural networks (6) with one single time-varying delay coupling. The obtained results can be applied to systems both with or without delays. With the one delay coupling, the synchronized state is different from those of a single node without coupling. Especially, the degree of the nodes and the delayed coupling matrix heavily influence the synchronized state. In addition, two neural networks are given to verify the effectiveness of the proposed criteria.

The remainder of this Letter is organized as follows: in Section 2, we present the problem formulations for global synchronization of system (6), some lemmas and notations that will be used throughout this Letter. In Section 3, global synchronization are discussed in two cases. In Section 4, several examples are given to show the validity of the obtained results. Finally, conclusions are drawn in Section 5.