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Output entanglement and squeezing of two-mode fields generated by a single atom

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ABSTRACT

A single four-level atom interacting with two-mode cavities is investigated. Under large detuning condition, we obtain the effective Hamiltonian which is unitary squeezing operator of two-mode fields. Employing the input–output theory, we find that the entanglement and squeezing of the output fields can be achieved. By analyzing the squeezing spectrum, we show that asymmetric detuning and asymmetric atomic initial state split the squeezing spectrum from one valley into two minimum values, and appropriate leakage of the cavity is needed for obtaining output entangled fields.

1. Introduction

One of the most intriguing features of quantum mechanics is entanglement, which has been recognized as a valuable resource for quantum information process. Discontinuous variables and continuous variables entanglement, as two kinds of entanglement resource, both have been concentrated much more attention. Continuous variables entanglement, compared with its partner discontinuous variables, has many advantages in quantum-information science [1] and can be used to efficiently implement quantum information process by utilizing the continuous quadrature variables of the quantized electromagnetic fields.

Conventionally, two-mode squeezed state emerging from the nonlinear optical interaction of a laser with a crystal (from parametric amplification or oscillation) is a typical continuous variables entanglement. Recently, it has been shown that correlated spontaneous emission laser can also work as continuous variables entanglement producer and amplifier [2–7]. Guzmán [8] proposed a method of generating unitary single and two-mode field squeezing in an optical cavity with an atomic cloud. As a result of realization of a single atom laser in experiment [9,10], people began to interest in generating two-mode entanglement via single-atom system [11–16]. Morigi et al. [11,12] have shown that a single trapped atom allows for the generation of entangled light under certain conditions. One of our authors Zhou [13] has proposed generating unitary two-mode field squeezing in a single three-level atom interacting dispersively with two classical fields inside a doubly resonant cavity, which can produce a macroscopic entangled light. Our group also proposed schemes to generate continuous variables entanglement in a single atom system [7,14]. Most recently, based on the same atomic level scheme as single-atom laser experiment [10], Kiffner [16] investigated a single atom system to generate a two-mode entangled laser via standard linear laser theory.

Although output entanglement and squeezing have been studied extensively in other system, the existence of a squeezing operator in the system which is similar to that of the single atom laser experiment [10] has never been exhibited before. In this Letter, we study a similar atomic level as the experiment in [10] (but with two-mode fields). However, there they studied one mode laser, here we concentrate on the output entanglement of the cavity. Under large detuning condition, we deduce unitary squeezing operator of two-mode fields. By means of the input–output theory, we show that entanglement and squeezing of two-mode fields can be achieved at the output. This Letter differ from [16] in these aspects: We use effective Hamiltonian method to obtain a squeezing field operator decoupled from the atomic degrees of freedom rather than by tracing the atomic degrees of freedom. Instead of studying intracavity fields, we show output entanglement.

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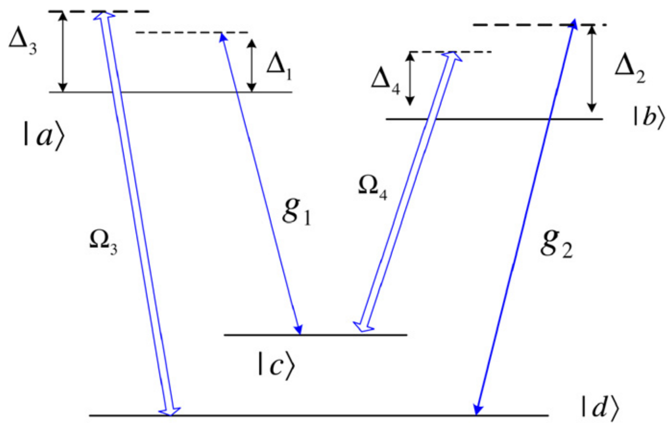

Fig. 1. The configuration of the atom. Two cavity modes interact with atomic transition $|a\rangle \leftrightarrow |c\rangle$ and $|b\rangle \leftrightarrow |d\rangle$ with detuning Δ_1 and Δ_2, respectively, while the two classical fields Ω_3 and Ω_4 drive the atomic level between $|a\rangle \leftrightarrow |d\rangle$ and $|b\rangle \leftrightarrow |c\rangle$ with detuning Δ_3 and Δ_4, respectively.

2. System description and calculations

We consider a single four-level atom trapped in a doubly resonant cavity, see Fig. 1. The atom interacts with two nondegenerate cavity modes. The first cavity mode couples to atomic transition $|a\rangle \leftrightarrow |c\rangle$ with the detuning Δ_1, and the second mode interacts with the atom on $|b\rangle \leftrightarrow |d\rangle$ with detuning Δ_2. The two classical laser fields with Rabi frequencies Ω_3 and Ω_4 drive the transitions $|a\rangle \leftrightarrow |d\rangle$ and $|b\rangle \leftrightarrow |c\rangle$ with detunings Δ_3 and Δ_4, respectively. The atomic configuration is the same as that in [16]. In the interaction picture, the Hamiltonian is

$$\begin{aligned} H_1 &= g_1 a_1 e^{-i\Delta_1 t}|a\rangle\langle c| + g_2 a_2 e^{-i\Delta_2 t}|b\rangle\langle d| \\ &\quad + \Omega_3 |a\rangle\langle d| e^{-i\Delta_3 t} + \Omega_4 |b\rangle\langle c| e^{-i\Delta_4 t} + \text{h.c.} \end{aligned} \tag{1}$$

Under large detuning condition $|\Delta_k| \gg \{|g_j|, |\Omega_l|\}$ ($k = 1, \ldots, 4$, $j = 1, 2$, $l = 3, 4$), we can adiabatically eliminate the excited level $|a\rangle$ and $|b\rangle$ and obtain the effective Hamiltonian

$$\begin{aligned} H_2 &= \left(\frac{|g_1|^2}{\Delta_1} a_1^\dagger a_1 + \frac{|\Omega_4|^2}{\Delta_4}\right)|c\rangle\langle c| \\ &\quad + \left(\frac{|g_2|^2}{\Delta_2} a_2^\dagger a_2 + \frac{|\Omega_3|^2}{\Delta_3}\right)|d\rangle\langle d| \\ &\quad + \left[\left(\frac{\Omega_3^* g_1}{\Delta_{13}} a_1 e^{i\delta_1 t} + \frac{\Omega_4 g_2^*}{\Delta_{24}} a_2^\dagger e^{-i\delta_2 t}\right)|d\rangle\langle c| + \text{h.c.}\right], \end{aligned} \tag{2}$$

where $\delta_1 = \Delta_3 - \Delta_1$, $\delta_2 = \Delta_4 - \Delta_2$, $\frac{1}{\Delta_{13}} = \frac{1}{2}(\frac{1}{\Delta_1} + \frac{1}{\Delta_3})$, $\frac{1}{\Delta_{24}} = \frac{1}{2}(\frac{1}{\Delta_2} + \frac{1}{\Delta_4})$. By making unitary transformation $U = e^{-itH_0}$ with

$$H_0 = \frac{|\Omega_4|^2}{\Delta_4}|c\rangle\langle c| + \frac{|\Omega_3|^2}{\Delta_3}|d\rangle\langle d| + \frac{\delta_1 + \delta_2}{2}\left(a_1^\dagger a_1 + a_2^\dagger a_2\right), \tag{3}$$

we have the new Hamiltonian

$$\begin{aligned} H_3 &= -\frac{\delta_1 + \delta_2}{2}\left(a_1^\dagger a_1 + a_2^\dagger a_2\right) \\ &\quad + \frac{|g_1|^2}{\Delta_1} a_1^\dagger a_1 |c\rangle\langle c| + \frac{|g_2|^2}{\Delta_2} a_2^\dagger a_2 |d\rangle\langle d| \\ &\quad + \left[\left(\frac{\Omega_3^* g_1}{\Delta_{13}} a_1 + \frac{\Omega_4 g_2^*}{\Delta_{24}} a_2^\dagger\right) e^{i\delta t}|d\rangle\langle c| + \text{h.c.}\right], \end{aligned} \tag{4}$$

where $\delta = \frac{|\Omega_3|^2}{\Delta_3} - \frac{|\Omega_4|^2}{\Delta_4} + \frac{\delta_1 - \delta_2}{2}$. If $|\delta| \gg \{|\frac{\Omega_3^* g_1}{\Delta_{13}}|, |\frac{\Omega_4 g_2^*}{\Delta_{24}}|\}$, we can perform adiabatic elimination once more and have

$$\begin{aligned} H_4 &= -\frac{\delta_1 + \delta_2}{2}\left(a_1^\dagger a_1 + a_2^\dagger a_2\right) \\ &\quad + \frac{|g_1|^2}{\Delta_1} a_1^\dagger a_1 |c\rangle\langle c| + \frac{|g_2|^2}{\Delta_2} a_2^\dagger a_2 |d\rangle\langle d| \\ &\quad + \frac{1}{\delta}\left[\left(\frac{\Omega_3^* g_1}{\Delta_{13}} a_1 + \frac{\Omega_4 g_2^*}{\Delta_{24}} a_2^\dagger\right)\left(\frac{\Omega_3 g_1^*}{\Delta_{13}} a_1^\dagger + \frac{\Omega_4^* g_2}{\Delta_{24}} a_2\right)|d\rangle\langle d| \right. \\ &\quad \left. - \left(\frac{\Omega_3 g_1^*}{\Delta_{13}} a_1^\dagger + \frac{\Omega_4^* g_2}{\Delta_{24}} a_2\right)\left(\frac{\Omega_3^* g_1}{\Delta_{13}} a_1 + \frac{\Omega_4 g_2^*}{\Delta_{24}} a_2^\dagger\right)|c\rangle\langle c|\right]. \end{aligned} \tag{5}$$

If the atom is initially in state $|d\rangle$, we finally have the effective Hamiltonian taken on the atomic state $|d\rangle$ as

$$H_{\text{eff}} = \lambda_1 a_1^\dagger a_1 + \lambda_2 a_2^\dagger a_2 + \eta a_1 a_2 + \eta^* a_1^\dagger a_2^\dagger, \tag{6}$$

with

$$\begin{aligned} \lambda_1 &= \frac{|\Omega_3 g_1|^2}{\delta \Delta_{13}^2} - \frac{\delta_1 + \delta_2}{2}, \\ \lambda_2 &= \frac{|\Omega_4 g_2|^2}{\delta \Delta_{24}^2} + \frac{|g_2|^2}{\Delta_2} - \frac{\delta_1 + \delta_2}{2}, \\ \eta &= \frac{g_1 g_2 \Omega_3^* \Omega_4^*}{\delta \Delta_{13} \Delta_{24}}. \end{aligned} \tag{7}$$

In Eq. (6), we have thrown off a constant which does not affect the dynamics of the system. Because the initial atomic state is $|d\rangle$, only the terms which take action on $|d\rangle$ survive. The stark shift $\frac{|g_1|^2}{\Delta_1} a_1^\dagger a_1 |c\rangle\langle c|$ has no contribution and $\frac{|g_2|^2}{\Delta_2} a_2^\dagger a_2 |d\rangle\langle d|$ remain (see the second line in Eq. (5)). Thus, λ_1 and λ_2 are asymmetric in form. We will show the effect of the asymmetry on the output squeezing and entanglement.

If the initial cavity fields are in coherent state $|\epsilon_1, \epsilon_2\rangle$ (with the help of two laser pumping, we can easy obtain the initial two-mode coherent state), we can use $SU(1,1)$ algebra to obtain evolution of wave function of the fields with $|\Psi_f(\tau)\rangle = e^{-iH_{\text{eff}}\tau}|\Psi_f(0)\rangle$. The exact expression of the fields evolution is a two-mode coherent-squeezed state as

$$\left|\Psi_f(\tau)\right\rangle = S(\vartheta)|\epsilon_1, \epsilon_2\rangle, \tag{8}$$

where $\vartheta = re^{i\varepsilon}$, and the squeeze parameter r (ε) is determined by $r = \tanh^{-1}|\tau\eta^* b_0 \sinh\phi|$ ($\tan\varepsilon = \text{Im}(-i\eta^* b_0 \sinh\phi)/\text{Re}(-i\eta^* b_0 \times \sinh\phi)$) with $\phi^2 = [|\eta|^2 - (\frac{\lambda_1+\lambda_2}{2})^2]\tau^2$, $b_0 = [\phi\cosh\phi + i\tau(\lambda_1 + \lambda_2)/2\sinh\phi]^{-1}$ [13]. The evolution time τ is limited by the $\tau_{\text{diss}} = \min(\frac{1}{\kappa_1}, \frac{1}{\kappa_2})$, where κ_1 and κ_2 are the decay rates of modes 1 and 2. So, the intensity of the fields cannot be increased into infinity with time evolution although the initial coherent state can effectively enhance the intensity of the cavity fields. Actually, the intensity of fields cannot be increased largely due to the loss of the cavity and the large detunings condition. Consequently, the adiabatic elimination still can be used within τ_{diss} only if the intensity of the quantum fields is not larger than the intensity of two classical fields Ω_3 and Ω_4. The decay effects will be discussed in next section where we do not need narrow the evolution time because physical quantities are automatically limited by time evolution after considering the decays. On the other hand, from Eq. (7) we see that the enhanced intensity of the fields do not affect the entanglement of the two modes because the entanglement results from the squeeze parameter.

3. Output squeezing and entanglement

We now concentrate on the squeezing properties of the outgoing cavity fields which can be detected and used as entanglement source. To evaluate the entangled light outside the cavity, we employ the input–output theory [17,18]. We assume that the two