



# A comprehensive multi-local-world model for complex networks

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## ABSTRACT

The nodes in a community within a network are much more connected to each other than to the others outside the community in the same network. This phenomenon has been commonly observed from many real-world networks, ranging from social to biological even to technical networks. Meanwhile, the number of communities in some real-world networks, such as the Internet and most social networks, are evolving with time. To model this kind of networks, the present Letter proposes a multi-local-world (MLW) model to capture and describe their essential topological properties. Based on the mean-field theory, the degree distribution of this model is obtained analytically, showing that the generated network has a novel topological feature as being not completely random nor completely scale-free but behaving somewhere between them. As a typical application, the MLW model is applied to characterize the Internet against some other models such as the BA, GBA, Fitness and HOT models, demonstrating the superiority of the new model.

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## 1. Introduction

Being within the human society today, one is often surprised to see that every individual is surrounded by many kinds of complex networks and even human themselves are parts of some complex networks.

Traditionally, networks with complex topologies are described by the random graph theory attributed to Erdős and Rényi (ER) [1]. Spurred by the increasing computing power of high-speed and grid computers as well as the availability of huge real databases in various scientific and humanity fields, some intensive computation with careful analysis on these empirical data have indicated that in many respects real-world complex networks are not completely random nor completely regular but behave somewhere in between them. Consequently, the public attention has shifted towards more realistic network models that can generate topologies with some properties consistent with the measurements and observations about the real world. In this endeavor, one of the most significant models of complex networks, the Barabási–Albert (BA) scale-free model [2], was developed. In this model, new nodes are being added and a fixed number of edges are being connected to the existing nodes with a preferential probability towards those nodes that already have large connections. The BA model can provide valuable insights into many real-world networks, including some social networks where the nodes represent individuals, or-

ganizations or countries, and the links characterize the social interactions among them; it also well models the WWW where the HTML documents are connected by hyperlinks pointing from one page to another; to name just a couple of examples. To date, based on different ideas and mechanisms, several similar network models [3–8] have been proposed to describe the common scale-free features of various complex networks.

Noticeably, however, in most network models that can generate scale-free features by means of preferential attachment, the mechanism responsible for the emergence of the scale-free topology of the network has a global nature. That is, the probability that an existing node receives a new link from the incoming node is with respect to all nodes and all links in the whole network. Clearly, this is not always the case in real life. A typical example in point is the World Trade Web (WTW): reportedly [9] the global preferential attachment mechanism does not work for those countries that have less than 20 trading connections with other countries in the world; but many countries are accelerating their economic cooperation in various regional economic-cooperative organizations such as EU, ASEAN, and NAFTA. This situation indicates that the preferential attachment mechanism actually only exists within local economic regions of the WTW. Another even more familiar example with local but not global preferential attachment is the Internet. In the Internet, due to the technical and economical limitations, a router in one autonomous system (AS) favors a connection of shorter or shortest path within the same AS when placing new links. As commonly known and indeed quite obvious, the Internet can be divided into many sub-networks, where the nodes in the same sub-network are tightly connected while the

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nodes in different sub-networks have very few connections. This is a prominent localization effect, yielding the community structures of the Internet and many other networks the like.

To capture the community feature of networks, Watts et al. [10] and Motter et al. [11] proposed some good network models, but with a fixed number of nodes therefore not evolving. As a result, these models fail to model many real networks that are growing and evolutionary. To fix the problem so as to describe the community feature of evolving networks, Grönlund and Holme [12] proposed a networked seceder model; Boguna et al. [13] further developed a network model based on a social distance attachment mechanism. Yet, in these two models, a node belonging to one community in the network may have no connections with any others within the same community but only with some nodes in other communities. More recently, Li and Maini [14] proposed a network model with the community feature based on two different preferential attachment mechanisms; followed by the study of [11] and the work of Xuan et al. [15] which suggests a model with both community and hierarchical structures. Moreover, by considering the effect of weights on networks, Fan et al. [16] developed a multi-community weight-driven bipartite network model. A common feature of these three models is that the network evolves with time but the number of communities in the network is always constant. However, recent research [17] has shown that the number of communities may also change as the network evolves. Furthermore, by taking into account the effects of the community structure on network properties and dynamics [18,19], one clearly needs a better network model that can precisely describe the topology of an evolving network with a community structure, generated under a localized preferential attachment mechanism.

In this Letter, a multi-local-world model is proposed to describe the topology of evolving networks with a varying number of communities. This model has two ingredients: a preferential attachment and a growth mechanism. Differing from the other existing models, however, these two mechanisms work only on a local restricted area, called “local-world” hereafter, in a way similar to the community structure but is generated by different means. Based on careful and detailed analysis, it is found that the degree distribution of the new model follows a novel law: it has an exponential form when the degree is below a critical threshold, beyond which it automatically follows a power-law. On the other hand, the number of communities in the network is changing with time, determined by a key parameter of the network. As a typical application, this MLW model is applied to describe the Internet topology. Compared with other models such as the BA model [2], the generalized BA (GBA) model [20], the HOT model [21] and Fitness model [22], our new MLW model is superior in the sense of better capturing the essential topological features of the real Internet.

## 2. The new multi-local-world model

The generating algorithm for the proposed MLW model is outlined as follows:

Initialization: Start with  $m$  ( $m \geq 1$ ) isolated local-worlds, with  $m_0$  ( $m_0 \geq 1$ ) isolated nodes in each local-world.

[For convenience, one may label each local-world by a unique identifier, e.g. a letter such as  $A, B, \dots$ , which is not always necessary].

Process: At each step, perform one of the following five operations:

- (i) With probability  $p$ , a new local-world is created, which contains  $m_0$  nodes and  $e_0$  links among these nodes. [Meanwhile, a unique identifier is generated to label this new local-world.]
- (ii) With probability  $q$ , a new node is added to an existing local-world, which brings  $m_1$  new links connecting to the nodes

within the same local-world. In doing so, first a local-world  $\Omega$  is selected at random, and then a node in this local-world  $\Omega$  is chosen with probability given by (1). The new incoming node is then connected to the selected node. This process is repeated  $m_1$  times. Here

$$\Pi(k_i) = \frac{k_i + \alpha}{\sum_{j \in \Omega} (k_j + \alpha)}, \quad (1)$$

in which  $\Omega$  means the  $\Omega$ th local-world where the chosen node  $i$  locates, and the parameter  $\alpha > 0$  represents the “attractiveness” of node  $i$ , which is used to govern the probability for the “young” nodes to receive new links.

- (iii) With probability  $r$ ,  $m_2$  links are added to within a chosen local-world. In doing so, first a local-world  $\Omega$  is selected at random, and then one end of a link is connected to a randomly chosen node while the other end of the link is connected to all the other nodes inside  $\Omega$  according to probability (1). This process is repeated  $m_2$  times.
- (iv) With probability  $s$ ,  $m_3$  links are deleted within a chosen local-world. In doing so, first a local-world  $\Omega$  is selected at random, and then one end of a link is randomly chosen while the other end of the link is selected according to probability (2). This process is repeated  $m_3$  times. Here

$$\Pi'(k_i) = \frac{1}{N_{\Omega}(t) - 1} (1 - \Pi(k_i)), \quad (2)$$

where  $N_{\Omega}(t)$  represents the number of nodes within the  $\Omega$ th local-world in the network at the current step  $t$ ,  $\Pi(k_i)$  is determined by (1), and the  $-1$  in the denominator means to exclude the selected node itself.

- (v) With probability  $u$ ,  $m_4$  links are added between different local-worlds. In doing so, first a local-world is randomly selected; then a node in this local-world is randomly chosen with probability (1), which is used as one end of a link while the other end of the link, which is in another randomly chosen local-world, is selected according to probability (1). This process is repeated  $m_4$  times.

In this model, the parameters satisfy  $0 \leq p, q, r, s, u \leq 1$ , and  $p + q + r + s + u = 1$ .

This ends the iterative algorithm.

Using the mean-field theory, one can obtain the degree distribution of the new model, as follows.

- (i) Creation of a new local-world with probability  $p$ .

At this step, the degree of node  $i$  in the existing local-world  $\Omega$  does not change over time since the original nodes in the newly created local-world have no links with any other nodes in the existing local-worlds. Thus, one has

$$\left( \frac{\partial k_i}{\partial t} \right)_{(i)} = 0, \quad (3)$$

where the subscript (i) indicates the step number, to avoid possible confusion with the same operation at different steps such as the next one.

- (ii) Addition of a new node joining the local-world  $\Omega$  with probability  $q$ .

$$\left( \frac{\partial k_i}{\partial t} \right)_{(ii)} = \frac{m_1 q}{m + t p} \Pi(k_i). \quad (4)$$

The term on the right-hand side corresponds to the random selection of a local-world, and a node is selected via preferential attachment specified by (1). Since there are  $m_1$  links between the new node and the existing nodes, the coefficient equals  $m_1$ .

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