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Feedback controlled dephasing and population relaxation in a two-level system

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ABSTRACT

This Letter presents the maximum achievable stability and purity that can be obtained in a two-level system with both dephasing and population relaxation processes by using homodyne-mediated feedback control. An analytic formula giving the optimal amplitudes of the driving and feedback for the steady-state is also presented. Experimental examples are used to show the importance of controlling the dephasing process.

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1. Introduction

A central issue in quantum information processing is quantum coherence, i.e., how long a quantum state survives without decay allowing robust quantum computation [1–3]. However, real quantum systems will unavoidably be influenced by surrounding environments which gives rise to decoherence processes [4]. One type of decoherence process is called population relaxation and the other is called dephasing. Population relaxation is a dissipative decoherence process. It is described by a changing of population inversion σ_z and results in energy loss. The dephasing process is a non-dissipative process. It comes from a randomization of the phases of the atomic wave functions by thermal and vacuum fluctuations in the electromagnetic field, causing the decay of the overlap of the upper and lower state wave function [5,6]. The population of a two-level atom will not be changed during the dephasing process, but the phase of the atomic dipole will be randomized [7], or more precisely the off-diagonal density matrix elements are destroyed. As a result, the density matrix becomes a statistical mixture and thus does not display any coherence effects.

Current approaches to decoherence control can be categorized as: quantum error correction (QEC) [8–10], decoherence-free subspace (DFS) [11–15], dynamical decoupling, and quantum feedback [16,17].

The QEC approach actively corrects quantum computational errors. It accomplishes this by using proper codewords to encode the state to be protected into carefully selected subspaces of the joint Hilbert space of the system and a number of ancillary systems [8–10]. The main limitation of the QEC approach for removing decoherence is the large number of extra qubits required to store the system state. For example, correcting all possible single qubit errors requires at least five qubits [18]. The number of extra qubits increases rapidly if fault tolerant error correction is realized [13].

A DFS is a passive quantum error avoiding method, in which no measurements or recovery operations are performed to detect and correct errors [11–15]. The basic idea is to encode the information into a region of the Hilbert space where the quantum information of the system is inaccessible by the environment. Experiments successfully implementing the DFS approach have been realized in linear optics [19,20], trapped ions [21] and nuclear magnetic resonance [22,23]. The DFS approach is only possible if the system-environment interaction has certain symmetries. Unfortunately, not all quantum systems have such a region in their Hilbert space. It is theoretically known that DFS can be utilized with QEC [24], the quantum Zeno effect [25] and dynamical decoupling [26].

The dynamical decoupling scheme uses repetitive pulses to remove some of the undesirable system-environment interactions to suppress decoherence [26–33]. This technique can significantly slow down decoherence, but is not able to remove the system entropy.

The quantum feedback approach represents the earliest method for decoherence control [16,17,34]. In this approach, the quantum system of interest is subjected to continuous photodetection and the information obtained from these measurements is used to achieve control of quantum dynamics and for the preparation of desired quantum states. Manipulating quantum systems such as atoms or trapped ions by feedback is not only of fundamental theoretical interest in quantum mechanics, but also opens up possibilities to generate various interesting quantum states in the laboratory [35,36].

In this Letter we examine how well homodyne-mediated feedback can control a two-level system accounting for an additional dephasing process. We have previously investigated feedback con-

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trol of the population relaxation process in a two-level system to achieve qubit stability [34]. However, for practical experiments it is often necessary to account for an additional non-dissipative dephasing process. The dephasing process may arise from elastic collisions in an atomic vapor, elastic phonon scattering in a solid, or photon shot noise in the measurement field [5,7,37], etc. The analysis of homodyne-mediated feedback control in which both dephasing and population relaxation processes are present thus makes our previous model more closely resemble an actual experimental realization.

Similar problems have been considered in a noisy qubit using tracking control method to maintain coherence [38]. A feedback stabilization of eigenstates of a continuously measured observable has been investigated in a higher-dimensional system [39]. The stabilization of two nonorthogonal states in a two-level system in a discreet time domain feedback scheme has also been recently studied [40].

This Letter shows that both the stability and purity of a two-level system is sensitive to the additional dephasing effect. This Letter also shows that feedback can stabilize the system state in both the upper and lower halves of the Bloch sphere in the presence of both dephasing and population relaxation. The stability of the optimal states in the upper and lower halves of the Bloch sphere are affected symmetrically by the dephasing rate. In this Letter an analytic solution for the steady-state is obtained. This leads to an analytic formula giving the optimal values of driving and feedback amplitude to maximize both stability and purity. The concluding section of this Letter discusses some experiments that may benefit from these results.

2. Feedback stabilization in the presence of dephasing and population relaxation

The system to be considered is shown in Fig. 1. The approach used in this investigation is called "Quantum Trajectories" [5,6]. We are interested in how well feedback can counter balance the effect of population relaxation with the addition of a dephasing process. We measure how well feedback works by finding the most stable pure state.

We model the dephasing process by considering the population inversion $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ coupled to a high temperature heat bath or vacuum fluctuations in the electromagnetic field, which can be described by an additional Hamiltonian [7]

$$H_z = \sigma_z \sqrt{\Gamma}\xi(t), \tag{1}$$

where Γ stands for the non-dissipative dephasing rate which will cause phase randomization. The term $\xi(t)$ represents Gaussian white noise [34],

By using the same method presented in Ref. [34], the homodyne-mediated feedback master equation including the non-dissipative dephasing process becomes

$$\dot{\rho} = \mathcal{L}\rho + \mathcal{K}\left(\sqrt{\gamma}\sigma\rho + \rho\sqrt{\gamma}\sigma^{\dagger}\right) + \frac{1}{2}\mathcal{K}^2\rho \tag{2}$$

where

$$\mathcal{L} = -i[\alpha\sigma_y, \rho] + \gamma\mathcal{D}[\sigma]\rho + \Gamma\mathcal{D}[\sigma_z]\rho. \tag{3}$$

The Liouville superoperator $\mathcal{K}$ gives reversible evolution with

$$\mathcal{K}\rho = -i[F, \rho]. \tag{4}$$

In Eqs. (2)–(4) ρ describes the system state, γ is the decay rate, $\sigma = |e\rangle\langle g|$ is the system lowering operator and the atom is driven by a resonant classical driving field with Rabi frequency 2α. The

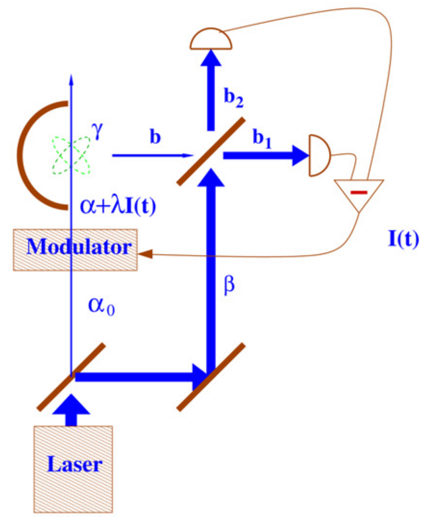

Fig. 1. Diagram of the experimental apparatus. The laser beam is split to produce both the local oscillator β and the field α_0 which is modulated using the measured homodyne photocurrent $I(t)$. The modulated beam, with amplitude proportional to $\alpha + \lambda I(t)$, drives an atom at the center of the parabolic mirror. Here λ is the feedback amplitude. The fluorescence thus collected is subject to homodyne detection using the local oscillator, and gives rise to the homodyne photocurrent $I(t)$.

Lindblad superoperator is defined as $\mathcal{D}[A]B \equiv ABA^{\dagger} - \{A^{\dagger}A, B\}/2$ [41]. The feedback Hamiltonian is given by $H_{\text{fb}} = \lambda\sigma_y I(t)$, where $F = \lambda\sigma_y$ is the feedback operator and λ feedback amplitude. $I(t)$ is the photocurrent given by:

$$I(t) = \sqrt{\gamma}\langle\sigma_x\rangle_c(t) + \xi(t). \tag{5}$$

The subscript c means conditioned.

The reason for choosing a homodyne-mediated measurement rather than a direct photodetection is because there exists an interference effect between beam b and the local oscillator β in the homodyne-mediated measurement, and it is this interference effect that leads to the deterministic part of the homodyne photocurrent proportional to $x_c = \langle\sigma_x\rangle_c$, where σ_x is the system x quadrature information. Obtaining system x quadrature information is essential for controlling the dynamics of the system state in the x–z plane by feedback. In the case of direct photodetection, the x quadrature information is not available.

In Eq. (2), feedback can be turned off by setting $F = 0$. In the steady-state limit, i.e., $\dot{\rho} = 0$, the following linear equations can be obtained from Eq. (2) using Bloch representation.

$$\begin{cases} (-\frac{\gamma}{4} - \sqrt{\gamma}\lambda - \Gamma - \lambda^2 - \lambda^2)x_s + (\alpha)z_s = 0, \\ (-\frac{\gamma}{4} - \Gamma)y_s + (\sqrt{\gamma} + \lambda)z_s = (-\sqrt{\gamma}), \\ (-\alpha)x + (\lambda)y_s + (-\frac{\gamma}{2} - \sqrt{\gamma}\lambda - \lambda^2)z_s = (\frac{\gamma}{2} + \sqrt{\gamma}\lambda). \end{cases} \tag{6}$$

The steady-state solution can be found analytically as follows

$$x_s = \frac{\alpha B}{D}, \qquad y_s = \frac{\alpha^2 A + C}{D}, \qquad z_s = \frac{E}{D}, \tag{7}$$

where

$$A = -\sqrt{\gamma},$$

$$B = \left(\frac{\gamma}{2} + \sqrt{\gamma}\lambda\right)\left(\frac{\gamma}{4} + \Gamma\right) - \sqrt{\gamma}\lambda,$$

$$C = \frac{\gamma}{2}\lambda\left(\frac{\gamma}{4} + \sqrt{\gamma}\lambda + \Gamma + \lambda^2\right),$$

$$E = \left(\frac{\gamma}{4} + \sqrt{\gamma}\lambda + \Gamma + \lambda^2\right)\left[\left(\frac{\gamma}{4} + \Gamma\right)\left(\frac{\gamma}{2} + \sqrt{\gamma}\lambda\right) - \sqrt{\gamma}\lambda\right],$$