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Directional property of radiation emitted from entangled atoms

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ABSTRACT

We investigate the directional property of photons emitted spontaneously from a partially-excited many-atom system. There exists a strong directional correlation between the emitted photons and the photons that have been absorbed by laser excitation and among all emitted photons themselves. Such a strong correlation arises from entanglement of W-type generated in the atomic system during the process of absorption and emission.

1. Introduction

In this Letter we investigate the directional property of photons emitted spontaneously from a many-atom system. While the directional property of spontaneously emitted photons is an old issue extensively investigated in the past, especially in connection with the problem of superradiance [1–3], two papers appeared recently in which this issue was viewed from a new angle, i.e., from the viewpoint of entanglement. Scully et al. [4] considered a situation in which a many-atom system is illuminated by a laser pulse and one photon is observed to be absorbed by the atoms. As no information on which atom has absorbed the photon is available, the atoms are in an entangled state. They showed that the photon emitted from the atomic system in such an entangled state is directed along the absorbed photon, even though the atoms have no dipole moment. They see this result as an interesting consequence of many particle entanglement in a dense medium. Eberly [5] considered a many-atom system in a product state. He studied a situation in which a collection of atoms are independently and identically prepared with each atom associated with an equal and small nonzero probability to be in its excited state. He showed that the emitted photon is still directed along the absorbed photon. He concluded that whether the atoms are in a highly quantum-mechanical entangled state or in a nearly classical product state makes no difference.

The purpose of this work is to clarify the role of quantum entanglement in a many-atom spontaneous emission process. A typical physical situation we have in mind is depicted in Fig. 1. A pulse of wave vector $\vec{k}_0$ illuminates a large number N ($N \gg 1$) of atoms, each prepared initially in its ground state $|g\rangle$. After the pulse passes through the atoms, it arrives at the detector D, which counts the number of photons and tells how many photons have been absorbed by the atoms. We assume that each of the N atoms is equally likely to absorb a photon and that the number n of the absorbed photons (or the number of atoms which absorb the photons and are driven to the excited state $|e\rangle$) is less than the total number N of the atoms ($n < N$). The state of the atoms after the exciting pulse has left the atoms is then given by

$$|\psi\rangle = \frac{1}{\sqrt{{}_N C_n}} \sum_{(j_1 j_2 \cdots j_n)} e^{i\vec{k}_0\cdot(\vec{R}_{j_1}+\vec{R}_{j_2}+\cdots+\vec{R}_{j_n})} \left|e_{j_1} e_{j_2} \cdots e_{j_n} g^{N-n}\right\rangle, \tag{1}$$

where the summation is to be performed over all possible ${}_N C_n$ (${}_N C_n = \frac{N!}{n!(N-n)!}$) ways of selecting n out of N atoms, $\vec{R}_j$ refers

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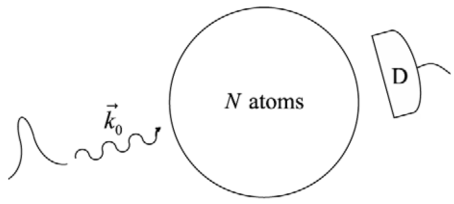

Fig. 1. Preparation of the many-atom entangled state. N atoms ($N \gg 1$) all in their ground state distributed randomly in space are illuminated by a pulse of wave vector $\vec{k}_0$. All atoms are equally likely to absorb a photon each. The pulse arrives at the detector D after interaction with the atoms. The detector D counts the number of photons absorbed by the atoms.

to the position of the jth atom, and $|e_{j_1}e_{j_2}\cdots e_{j_n}g^{N-n}\rangle$ indicates that the j_1th, j_2th, ..., and j_nth atoms are in their excited state $|e\rangle$, and the remaining $(N-n)$ atoms are in their ground state $|g\rangle$. The initial state (1) characterizes the situation we denote by (N,n) which signifies that there are N atoms in the system having n units of excitation transferred from the exciting pulse. A total of n photons will be emitted from the N-atom system in state (1), and we ask along which direction these n photons will be emitted. We ask, in particular, whether these emitted photons are directionally correlated with the absorbed photons. Throughout our discussion we assume that the atoms are distributed randomly in a region large compared with the wavelength of the emitted radiation.

We note that, as long as $0<n<N$, the state $|\psi\rangle$ of Eq. (1) is given in a linear superposition of states consisting of n excited atoms and $(N-n)$ unexcited atoms. In other words, it represents an entangled state of the type called the W state [6]. This particular type of entangled state is generated in the atomic system of Fig. 1, because one does not know which n atoms out of the N atoms have absorbed the photons from the pulse.

2. Two atoms

As a preliminary to the many-atom problem, we consider the case $(N=2, n=1)$ of two atoms having one unit of excitation transferred from an exciting pulse of wave vector $\vec{k}_0$. Taking $N=2$ and $n=1$ in Eq. (1), we have, for the initial atomic state,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(e^{i\vec{k}_0\cdot\vec{R}_1}|e_1g_2\rangle + e^{i\vec{k}_0\cdot\vec{R}_2}|g_1e_2\rangle\right). \tag{2}$$

One photon will be emitted from the two-atom system in state (2). Denoting the wave vector of the emitted photon as $\vec{k}$, the final state of the system of the two atoms and the radiation field is written as $|g_1g_21_{\vec{k}}\rangle$. The wave function of the system at an arbitrary time $t>0$ is given by

$$\begin{aligned}\left|\psi(t)\right\rangle &= \alpha_1(t)e^{-i\omega_a t}|e_1g_20\rangle + \alpha_2(t)e^{-i\omega_a t}|g_1e_20\rangle \\ &\quad + \int d^3\vec{k}\,\alpha_{\vec{k}}(t)e^{-i\omega_k t}|g_1g_21_{\vec{k}}\rangle,\end{aligned} \tag{3}$$

where ω_a is the atomic angular frequency ($\omega_a = \frac{E_e-E_g}{\hbar}$) and ω_k is the angular frequency of the emitted radiation ($\omega_k = c|\vec{k}| = ck$). We have inserted "0" to the atomic states $|e_1g_2\rangle$ and $|g_1e_2\rangle$ in Eq. (3) to emphasize that, when the atoms are in state $|e_1g_2\rangle$ or $|g_1e_2\rangle$, there is no photon in the radiation field. In writing Eq. (3), we have made the Weisskopf–Wigner approximation [7] limiting our consideration only to the states which can be reached from the initial state in accordance with the principle that the creation and annihilation of a photon are accompanied by the downward and upward transitions, respectively, of the atom. This approximation is equivalent to the rotating wave approximation. Substituting Eq. (3) into the time-dependent Schrödinger equation, we obtain

$$i\frac{d\alpha_1(t)}{dt} = \int d^3\vec{k}\, g^*_{\vec{k}} e^{i\vec{k}\cdot\vec{R}_1} e^{i(\omega_a-\omega_k)t}\alpha_{\vec{k}}(t), \tag{4a}$$

$$i\frac{d\alpha_2(t)}{dt} = \int d^3\vec{k}\, g^*_{\vec{k}} e^{i\vec{k}\cdot\vec{R}_2} e^{i(\omega_a-\omega_k)t}\alpha_{\vec{k}}(t), \tag{4b}$$

$$\begin{aligned} i\frac{d\alpha_{\vec{k}}(t)}{dt} &= g_{\vec{k}} e^{-i\vec{k}\cdot\vec{R}_1} e^{-i(\omega_a-\omega_k)t}\alpha_1(t) \\ &\quad + g_{\vec{k}} e^{-i\vec{k}\cdot\vec{R}_2} e^{-i(\omega_a-\omega_k)t}\alpha_2(t),\end{aligned} \tag{4c}$$

where $g_{\vec{k}}$ is the atom–field dipole coupling constant given by

$$g_{\vec{k}} = \frac{1}{\hbar}\langle g1_{\vec{k}}|H'|e0\rangle, \tag{5}$$

and H' is the interaction Hamiltonian ($H' = -\frac{e}{m}\vec{A}\cdot\vec{P}$). Eqs. (4) can be solved for the probability amplitudes $\alpha_1(t)$, $\alpha_2(t)$ and $\alpha_{\vec{k}}(t)$, subject to the initial condition $\alpha_1(0) = \frac{1}{\sqrt{2}}e^{i\vec{k}_0\cdot\vec{R}_1}$, $\alpha_2(0) = \frac{1}{\sqrt{2}}e^{i\vec{k}_0\cdot\vec{R}_2}$, $\alpha_{\vec{k}}(0)=0$, using the technique of Laplace transform or equivalent methods. For detailed procedure, we refer the readers to Refs. [2,8–10]. From the solution of Eqs. (4), we obtain the probability $P_{\vec{k}}(t\to\infty) = |\alpha_{\vec{k}}(t\to\infty)|^2$ of having a photon of wave vector $\vec{k}$ in the radiation field long after the state (2) was prepared, which reads

$$P_{\vec{k}}(t\to\infty) = |g_{\vec{k}}|^2\xi_k\left\{1+\cos\left[(\vec{k}-\vec{k}_0)\cdot\vec{R}\right] + D_{\vec{k}}\right\}, \tag{6}$$

where

$$\begin{aligned} D_{\vec{k}} &= \frac{\gamma_{12}^2}{(\omega_a-\omega_k)^2+\gamma^2}\left\{1+\cos\left[(\vec{k}+\vec{k}_0)\cdot\vec{R}\right]\right\} \\ &\quad - \frac{2\gamma_{12}\gamma}{(\omega_a-\omega_k)^2+\gamma^2}\left[\cos(\vec{k}_0\cdot\vec{R}) + \cos(\vec{k}\cdot\vec{R})\right],\end{aligned} \tag{7}$$

ξ_k is a real function depending on the magnitude but not on the direction of $\vec{k}$, given as

$$\xi_k = \frac{(\omega_a-\omega_k)^2+\gamma^2}{[(\omega_a-\omega_k)^2+(\gamma+\gamma_{12})^2][(\omega_a-\omega_k)^2+(\gamma-\gamma_{12})^2]}, \tag{8}$$

γ is the amplitude decay rate of the excited state of a single isolated atom, given by

$$\gamma = \frac{V}{(2\pi)^3}\frac{\omega_a^2}{c^3}\int d\Omega_k\left[|g_{\vec{k}}|^2\right]_{k=\frac{\omega_a}{c}}, \tag{9}$$

γ_{12} represents the part of the amplitude decay rate modified due to the interaction between the two atoms, in such a way that $(\gamma+\gamma_{12})$ and $(\gamma-\gamma_{12})$ are the decay rates of the symmetric $[\frac{1}{\sqrt{2}}(|e_1g_2\rangle + |g_1e_2\rangle)]$ and antisymmetric $[\frac{1}{\sqrt{2}}(|e_1g_2\rangle - |g_1e_2\rangle)]$ states, respectively,

$$\gamma_{12} = \frac{V}{(2\pi)^3}\frac{\omega_a^2}{c^3}\int d\Omega_k\left[|g_{\vec{k}}|^2 e^{i\vec{k}\cdot\vec{R}}\right]_{k=\frac{\omega_a}{c}}, \tag{10}$$

and $\vec{R} = \vec{R}_1 - \vec{R}_2$. The term $D_{\vec{k}}$ given by Eq. (7) represents "dynamical effects" arising from interaction between the two atoms, i.e., effects due to repeated absorption and emission as the radiation emitted from one atom can reach and excite the other. The complicated but interesting situation in which R is smaller than the radiation wavelength ($kR \lesssim 1$) and consequently these dynamical effects play an important role has been treated in detail in our earlier publication [11]. Here we limit ourselves to the situation where R is sufficiently large ($kR \gg 1$) that the contribution from the dynamical effects is relatively small. This allows one to look at the bare essence of the mechanism that determines the directional property of the spontaneously emitted photons without complications arising from the direct atom–atom interaction.

In the limit $kR \gg 1$ (or equivalently $\gamma_{12} \ll \gamma$), Eq. (6) becomes [9,10,12]

$$P_{\vec{k}}(\infty) \propto |g_{\vec{k}}|^2\left\{1+\cos\left[(\vec{k}-\vec{k}_0)\cdot\vec{R}\right]\right\}, \tag{11}$$