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Narrow transmission bands of quasi-1D comb-like photonic waveguides containing negative index materials

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ABSTRACT

Using the interface response theory, we investigate the band structure and transmission of quasi-1D comb-like photonic waveguides with side branches composed of negative index materials. Numerical results exhibit the existence of discrete modes in band structure. These discrete modes are corresponding to narrow transmission bands which separated by large forbidden band in the transmission spectrum. Meanwhile it is shown that the narrow transmission bands become narrower with the increase of the number of side branches. The above properties are still maintained when the negative index materials are dispersive and lossy.

1. Introduction

Negative index materials (NIMs) with both negative permittivity ε and permeability μ, proposed by Veslago in 1968 [1], have attracted tremendous attentions because of their unique electromagnetic properties in the last few years [2–7]. Many interesting characteristics have been proposed in photonic crystals with NIMs. Recently, people have started to study the band structure of one-dimensional photonic crystals composed of negative index materials and positive index materials (PIMs) [8–17]. Potential applications, such as narrow-frequency filtering [8,11–14], omnidirectional gap [15,16], have been proposed. Besides these, superluminal tunneling [18], beam shaping [19] and negative Hartmann effect [20] were studied in 1DPC consisting of NIMs and PIMs.

The quasi-1D comb-like photonic waveguide was proposed by Vasseur et al. [21,22]. The structure exhibits photonic band gaps, which is analogous to 1DPC, originates from the period of the system and the resonance states of the side branches. Recently, the negative index materials are introduced in the comb-like structure [23,24]. A special gap which is insensitive to the geometrical scaling and disorder was found. The broadening of the gap can be realized with much freedom [24]. In our present Letter, quasi-1D comb-like photonic waveguides with several side branches composed of negative index materials are studied. By analyzing the dispersion equation, we find that some discrete modes exist in band structure. These discrete modes are corresponding to narrow transmission bands in the transmission spectrum. Moreover, the narrow transmission band can be narrower by increasing the number of side branches. A non-ideal model when the negative index materials are dispersive and lossy is used to verify theses properties. These findings will lead to potential applications.

This outline of this Letter is as follows. In Section 2, we briefly introduce the theoretical model and give the analytic expressions for the band structure of the one-dimensional comb-like photonic waveguide and for the transmission coefficient through such a system. Section 3 presents the numerical results and analysis. Conclusions on this work are drawn in Section 4.

2. Theory

Let us consider a quasi-1D comb-like photonic waveguide containing negative index materials. The system is composed of an infinite one-dimensional backbone waveguide (made of medium 1 with positive index) along which N' identical side branches of length d_2 (composed of medium 2 with negative index) are grafted at N nodes distant from each other by a length d_1. N' and N are integers. Each medium j ($j=1$ for the backbone waveguide and $j=2$ for the side branches) is characterized by its permittivity ε_j and permeability μ_j. The geometry considered here is shown in Fig. 1.

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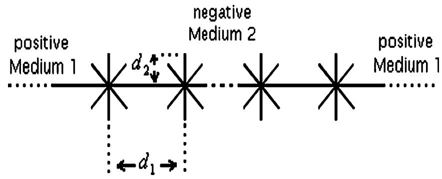

Fig. 1. Schematic representation of the quasi-1D comb-like photonic waveguides.

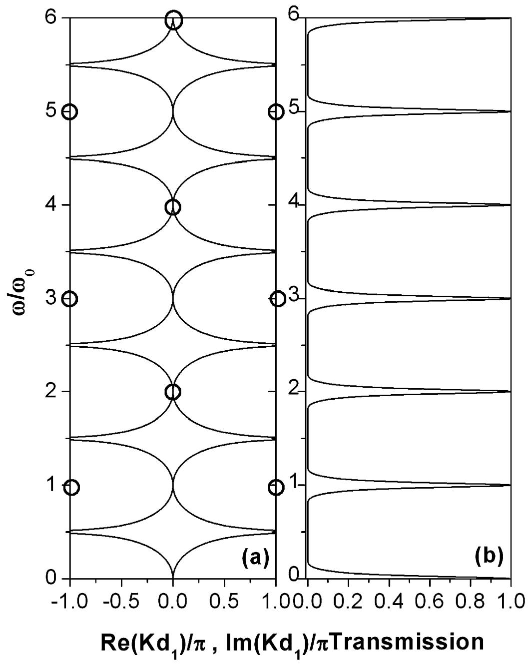

Fig. 2. (a) Dispersion relationship of one-dimensional structure depicted in Fig. 1 with $N \to \infty$, $N' = 1$ and the boundary condition $H = 0$. The circles shows the real solutions for K. The real solutions only have some discrete values at $K = 0$ and $K = \pm\pi/d_1$. The solid lines indicate the imaginary of K; (b) Transmission coefficient through the same structure with $N = 20$ nodes.

With the help of interface response theory, the dispersion relationship of the band structure for the infinite comb-like waveguide ($N \to \infty$) depicted in Fig. 1 can be written explicitly as [21,23]

$$\cos(Kd_1) = \cos(k_1 d_1) - \frac{N'}{2}\frac{Z_1}{Z_2}\sin(k_1 d_1)\tan(k_2 d_2), \qquad (1)$$

where $k_j = \delta\frac{\omega}{c}\sqrt{\varepsilon_j \mu_j}$, $Z_j = \sqrt{\frac{\mu_j}{\varepsilon_j}}$ ($j = 1, 2$; $\delta = \pm 1$) for the boundary conditions $H = 0$, namely, the vanishing magnetic field at the free extremities of side branches. We select $\delta = +1$ for positive index materials, $\delta = -1$ for negative index materials. In Eq. (1), K is the wave vector for propagation along the waveguide, which can be determined at each frequency ω. If the absolute value of the right side of Eq. (1) is smaller than 1, one can find real solution for K, the corresponding wave can propagates along the waveguide and belongs to the pass-band. Otherwise, K is a complex number, the wave cannot propagate and ω belongs to gaps of the comb-like waveguide.

In the case of a finite number N of nodes along the waveguide, the transmission coefficient for an incident wave is given by [21–23,25]

$$T = \left|\frac{2\sin(k_1 d_1)(t^2 - 1)t^N}{(1 - te^{ik_1 d_1})^2 - t^{2N}(t - e^{ik_1 d_1})^2}\right|^2 \qquad (2)$$

where N is the number of nodes, $t = e^{iKd_1}$ and K is determined by Eq. (1).

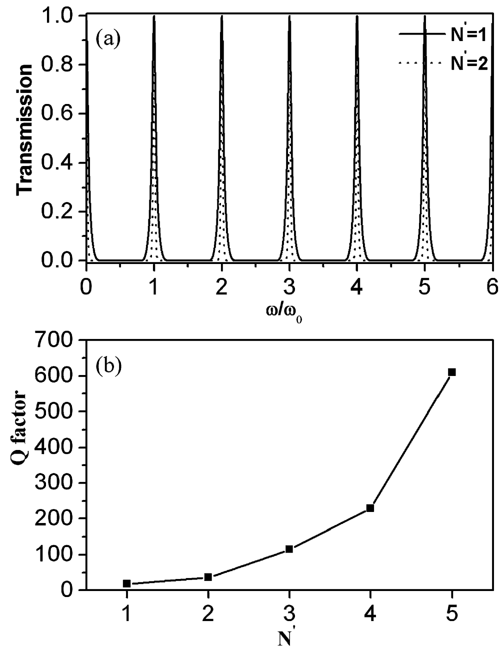

Fig. 3. (a) Transmission coefficient through the same structure with different number of side branches. The solid line is for $N' = 1$ and the dotted line is for $N' = 2$. (b) Variance of Q factor corresponding to the first transmittance mode $\omega/\omega_0 = 1$ with different number of side branches.

3. Numerical results and analysis

In this part, we will discuss our numerical results on dispersion relationship and transmission according to the above theory. We shall study the effect of the introduction of negative index materials on band structure and transmission coefficient. In Fig. 2(a), we present the dispersion curves of the photonic band structure for comb-like photonic waveguide depicted in Fig. 1 with $N \to \infty$, $N' = 1$ and the boundary conditions $H = 0$. The plot is given in terms of the reduced frequency ω/ω_0 ($\omega_0 = c\pi/d_1\sqrt{\varepsilon_1\mu_1}$) versus the reduced wave vector Kd_1. In our calculation, we choose $\varepsilon_2 = -\varepsilon_1$, $\mu_2 = -\mu_1$, and $d_1 = d_2$. The most interesting feature of the band structure is these discrete real solutions for K at the reduced frequency $\omega/\omega_0 = n$ ($n = 1, 2, 3, \ldots$). Fig. 2(b) demonstrates transmission spectral of such a structure with a finite number of nodes, namely, $N = 20$. It is obvious that there are some narrow-band transmission peaks at the reduced frequency $\omega/\omega_0 = n$ ($n = 1, 2, 3, \ldots$). At other reduced frequencies which correspond to the band gap, the transmission are forbidden. By virtue of the unique properties, the comb-like photonic waveguide containing negative index materials can be used as narrow-band filter.

The interesting phenomena can be explained by analyzing Eq. (1). If $\varepsilon_2 = -\varepsilon_1$, $\mu_2 = -\mu_1$, and $d_1 = d_2$, the dispersion relationship described by Eq. (1) can be simplify as

$$\cos(Kd_1) = \begin{cases} (1 + \cos^2\Omega)/2\cos\Omega, & N' = 1, \\ [2 + (N' - 2)\sin^2\Omega]/2\cos\Omega, & N' \geqslant 2. \end{cases} \qquad (3)$$

Where $\Omega = \omega\sqrt{\varepsilon_1\mu_1}d_1/c = \pi\omega/\omega_0$ and $\omega_0 = c\pi/d_1\sqrt{\varepsilon_1\mu_1}$. Through simple mathematic analysis, one can find that the absolute values of the right side of Eq. (3) is always larger than 1 except when $\Omega = n\pi$ for any N', that is, when the reduced frequency $\omega/\omega_0 = n$ ($n = 1, 2, 3, \ldots$). Therefore the band structure has discrete real solutions for K and the electromagnetic wave can transmit only at these points mentioned above. Keeping other parameters unchanged, the transmission through such a system for larger N' is shown in Fig. 3(a). We can find that the transmission peaks become narrower with the increasing number of side